University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Calculus I Fall 2009

Worksheet 4(Continuity)(Answers)

1. Determine whether or not the function given in each case is continuous at the given point. Give reasons for your answer.

a. $f(x) = \begin{cases} x^3 + x^2 & x \le -2 \\ 2x^2 - 4 & x > -2 \end{cases}$ at x = -2.

f is not continuous at x = -2 since
$\lim_{x \to -2} f(x)$ does not exist.

b. $f(x) = \begin{cases} |x^2 - 4| & -2 \le x \le 2 \\ 2x - 4 & x > 2 \\ 3x + 4 & x < -2 \end{cases}$ at x = 2 and at x = -2.

f is continuous at x = 2.
f is not continuous at x = - 2 since
$\lim_{x \to -2} f(x)$ does not exist.

c. $f(x) = \begin{cases} \dfrac{x^3 - 9x}{x^2 + x - 12} & x > 3 \\ \dfrac{10}{7} & x = 3 \\ \dfrac{2x^2}{7} & x < 3 \end{cases}$ at x = 3.

f is not continuous at x = 3 since
$\lim_{x \to 3} f(x) \ne f(3)$

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d. $f(x) = \begin{cases} x + \dfrac{1}{x} & x < 0 \\ -x^3 & x \geq 0 \end{cases}$ at x = 0.

f is not continuous at x = 0 since $\lim_{x \to 0} f(x)$ does not exist.

e. $f(x) = \begin{cases} x + \dfrac{1}{x} & x < 0 \\ -2 & x = 0 \\ -\dfrac{1}{x^3} & x > 0 \end{cases}$ at x = 0.

f is not continuous at x = 0.

2. In each case determine values of a so that the function given is continuous.

a. $f(x) = \begin{cases} 3x^3 - 4x^2 + a & x \leq -2 \\ 4x^2 - 1 & x > -2 \end{cases}$

For f to be continuous, a = 55

b. $f(x) = \begin{cases} \dfrac{x^3 + x^2 - ax}{x^2 - 1} & x \leq -2 \\ 2x^2 + 3x - 4 & x > -2 \end{cases}$

For f to be continuous, a = -1

3. Determine values of *a* and *b* so that the function given is continuous.

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$$f(x) = \begin{cases} 3x^3 - bx^2 + a & x < -2 \\ 1 & x = -2 \\ ax^2 + bx - 1 & x > -2 \end{cases}$$

For f to be continuous, a = -3 and b = -7.

4. Show that the cubic equation
$$x^3 + x^2 - x - 4 = 0$$
has a root in the interval (1,2).

5. If $f(x) = x^3 + x - 1$,
show that f has a zero between x = 0 and x = 1

6. Show that $f(x) = x^3 - 15x + 1$
has at least three zeros in the closed interval [-4, 4].