

1. Determine the equation of a straight line in each case:

a. The straight line passes through the point A(2,1) and has a slope of -2.

Answer:

The straight line has equation $y - 1 = -2(x - 2)$.

b. The straight line contains the points A(3,-4) and B(-1,2).

Answer:

The straight line has equation $y + 4 = -\frac{3}{2}(x - 3)$

c. The straight line has y-intercept = 5 and x-intercept = -3.

Answer:

The straight line has equation $y = \frac{5}{3}x + 5$

d. The straight line is parallel to the straight line $3x - 4y = 12$ and passes through the point (-2,-3).

Answer:

The straight line has equation $y + 3 = \frac{3}{4}(x + 2)$

e. The straight line is perpendicular to the straight line $4x + 5y = -20$ and passes through the mid-point of the line segment AB where A has coordinates (-1,1) and B has coordinates (5,-2).

Answer:

The straight line has equation $y + \frac{1}{2} = \frac{5}{4}(x - 2)$

2. Determine the equation of the circle which has diameter AB where A and B are the points given in 1(e).

Answer:

The circle has equation $(x - 2)^2 + \left(y + \frac{1}{2}\right)^2 = 45$

3. Determine the equation of the circle which has centre at $(-2,1)$ and which passes through the point $(2,4)$.

Answer:

The circle has equation $(x + 2)^2 + (y - 1)^2 = 25$

4. Determine the equation of the circle whose centre is at the point of intersection of the lines $2x - 3y = 7$ and $3x + 5y = 1$, and which has a radius of 4 units.

Answer:

The circle has equation $(x - 2)^2 + (y + 1)^2 = 16$

5. Determine the equation of the circle which is tangent to the x-axis and which has centre at the point $(3,-1)$.

Answer:

The circle has equation $(x - 3)^2 + (y + 1)^2 = 1$

6. Determine the equation of the circle which is tangent to the y-axis and which has centre at the point $(4,-2)$.

Answer:

The circle has equation $(x - 4)^2 + (y + 2)^2 = 16$

7. Determine the perpendicular distance from a point $P(h, k)$ to the straight line with equation $ax + by + c = 0$.

Answer:

The perpendicular distance, d , is given by $d = \frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

8. Determine the equation of the circle which has centre the point $C(2,3)$ and radius 4 units.

Answer:

The circle has equation $(x - 2)^2 + (y - 3)^2 = 16$

9. Determine the equation of the circle which passes through the point $A(3,-1)$ and has centre at $C(1,2)$.

Answer:

The circle has equation $(x - 1)^2 + (y - 2)^2 = 13$

10. Determine the equation of the circle which is tangent to the straight line $y = 7$ and which has centre at the point $C(2,1)$.

Answer:

The circle has equation $(x - 2)^2 + (y - 1)^2 = 36$

11. Determine the equation of the circle which is tangent to the straight line $3x + 4y = 12$ and which has centre at the point $(1,1)$.

Answer:

The circle has equation $(x - 1)^2 + (y - 1)^2 = 1$

12. The straight lines L_1 and L_2 have slopes equal to m_1 and m_2 respectively. If the straight lines are perpendicular, neither line being parallel to either axis, show that $m_1 m_2 = -1$. [Hint: You may use the fact that the slope of a straight line is the tangent of the angle that the line makes with the positive direction of the x-axis.]

13. Show that the equation of the tangent line to the circle with equation $x^2 + y^2 = r^2$, at the point (h,k) on the circle is given by $hx + ky = r^2$.

14. A circle has equation $x^2 + y^2 + 2gx + 2fy + c = 0$. The point $P(h,k)$ is on the circle. Show that the equation of the tangent line to the circle at P is given by:
 $hx + ky + g(x + h) + f(y + k) + c = 0$

Does this generalize to other conics?