Problem 1. (a) $K = [A|B] = \left[\begin{array}{cccc|c} 3 & -5 & -7 & -3 & 9 \\ 2 & -5 & 2 & -7 & 11 \\ -1 & 2 & 1 & 2 & -4 \end{array}\right]$

(b) $[K|I_3] \longrightarrow [R|U]$:
$R = \left[\begin{array}{cccc|c} 1 & 0 & -9 & 4 & -2 \\ 0 & 1 & -4 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$; $U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & -2 \\ 1 & 1 & 5 \end{bmatrix}$, ($U$ is not unique, e.g., $U = \begin{bmatrix} 0 & -2 & -5 \\ 0 & -1 & -2 \\ 1 & 1 & 5 \end{bmatrix}$, or a different one).

(c) Gen. sol. to the syst.: $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$; Particular solution: $\begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \end{bmatrix}$; Gen. sol. to the associated hom. syst.: $X_0 = t \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$; Basic solutions: $\begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

Problem 2. (a) $\begin{bmatrix} 7 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$; $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.
($x = -1$; $y = 2$).

(b) $A = \begin{bmatrix} -2 & 5 \\ 3 & -7 \end{bmatrix}$.

Problem 3. $A^{-1} = \begin{bmatrix} -3 & 5 & -1 \\ 4 & -5 & -1 \\ 1 & -2 & 1 \end{bmatrix}$.

Problem 4. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. (This product is not unique.)

Problem 5. (a) $a = x + 2y$ and $b = x + y$;

(b) $T\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = (x + 2y) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + (x + y) \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2x + 3y \\ 5x + 7y \end{bmatrix}$;

(c) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$;

(d) $A_T = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$;

(e) Yes (T is invertible). $A_{T^{-1}} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

Problem 6. (a) $S_0 = [1\ 0\ 0]^T$, $S_2 = PS_0$, $S_2 = PS_1 = PPS_0 = P^2 S_0 = [16/25\ \mathbf{9/25}]^T$. Therefore, the probability to be in state 2 after 2 transitions is 9/25.

(b) Since $P^1 = P$ has no zero entries, the chain is regular. Its steady state-vector S is the only probability vector which solves the homogeneous system $(I_2 - P)X = 0$. $S = [4/7\ 3/7]^T$.