1. Show that the general solution of the Laplace equation in a disk with no θ dependence
$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} = 0$$
is $u = c\ln(kr)$ with $k > 0$.

2. A thin homogeneous annular disk with inner radius R_1 and outer radius R_2 has inner boundary temperature 0° and outer boundary temperature 100°. Solve the steady state heat equation with these boundary conditions. At what points in the disk will the temperature be 50°?

   What kind of mean is this?

3. Consider the Euler differential equation
$$ar^2R'' + brR' + cR = 0$$
for constants a, b, c and $R = R(r)$. Assuming a solution of the form $R(r) = r^k$, what equation does k satisfy?

4. Consider the surface of revolution obtained by revolving the circle $x^2 + y^2 = 1$ about the x-axis. Let $-1 \le a < b \le 1$. Compute the surface area of the sphere of the band between $x = a$ and $x = b$. Show that this is equal to the surface area of the circumscribed cylinder of radius 1 in the band between $x = a$ and $x = b$. This is an explanation of why the zonal harmonics are actually orthogonal on the sphere since the Legendre polynomials are on the interval. In other words, the area measure on the sphere that you integrate with respect to for the zonal harmonics is proportional to the area measure on the cylinder, which is proportional to the linear measure on a straight line!

   This theorem about the cylinder and the sphere is originally due to the greatest mathematician of antiquity, Archimedes.