

1. Let R be the square $0 \leq x \leq 1$, $0 \leq y \leq 1$. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ in the region R if the boundary conditions are $u(x, 0) = 0$, $u(0, y) = 0$, $u(1, y) = y$, $u(x, 1) = x$.

2. Find two linearly independent solutions on the positive x -axis for the equation

$$xy'' + (1 - x)y' + 3y = 0.$$

3. (a) Given that $\Gamma(1/2) = \sqrt{\pi}$, find the value of $\Gamma(n/2)$, n an integer.
(b) Show that

$$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x \quad \text{and} \quad J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x.$$

Hint: just expand the series.

- (c) One of the many identities that the Bessel functions satisfy is

$$xJ_{p+1} - 2pJ_p + xJ_{p-1} = 0.$$

Find a simple expression for $J_{5/2}(x)$. Hint: first find $J_{3/2}(x)$.

4. (Extra for experts.) Establish the identity in 3(c).