

1. Solve the first order equation $5u_x + 42u_y = 0$ if $u(x, 0) = e^{-x}$.
2. Find the characteristics of $(1 + x)u_x - (1 - y)u_y = 0$.
3. Solve the differential equation for the function $y = y(x)$ if

$$(1 + x^2)y' + 3y = 0, \quad y(0) = 1.$$

4. Find the general solution of the differential equation

$$y''' + 5y'' - 8y' - 12y = 0.$$

5. Find the convolution $H(t)e^{-at} * H(t)e^{-bt}$ where a and b are positive constants and $H(t)$ is the Heaviside unit step function.
6. Consider the inner product on polynomials p, q given by

$$\int_0^1 p(x)q(x) dx.$$

Orthogonalize the vectors $1, x - 1, (x - 1)^2$.

7. Find the Fourier series of $f(x) = \cos^3 x$, thinking of f as defined on the interval $[-\pi, \pi]$.
8. Find the eigenvectors of the linear operator L with matrix

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

9. Let L be the symmetric linear transformation defined by the matrix

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

Show that L has two distinct real eigenvalues except in the trivial case $b = 0$ and $a = c$.

10. Find all eigenvalues and eigenvectors for the Sturm-Liouville system

$$y'' + \lambda y = 0, \quad y'(-\pi) = 0, \quad y'(\pi) = 0.$$

11. Find the solution of the one dimensional wave equation on the interval $[0, \pi]$ subject to the endpoint conditions $u(0, t) = u(\pi, t) = 0$ if the initial conditions are

$$u(x, 0) = \sin nx, \quad u_t(x, 0) \equiv 0.$$