Post-quantum cryptography

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University of Illinois at Chicago

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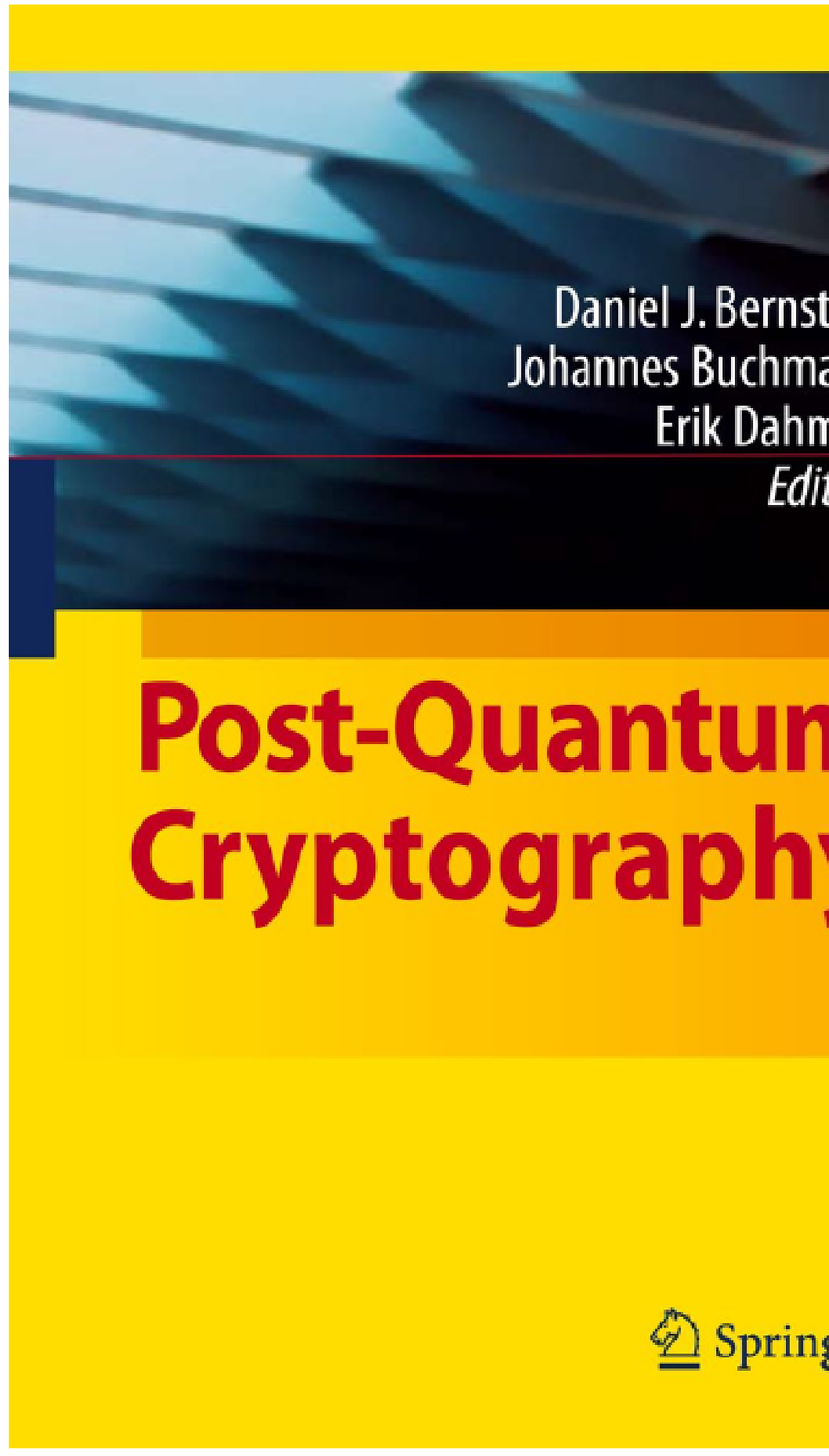

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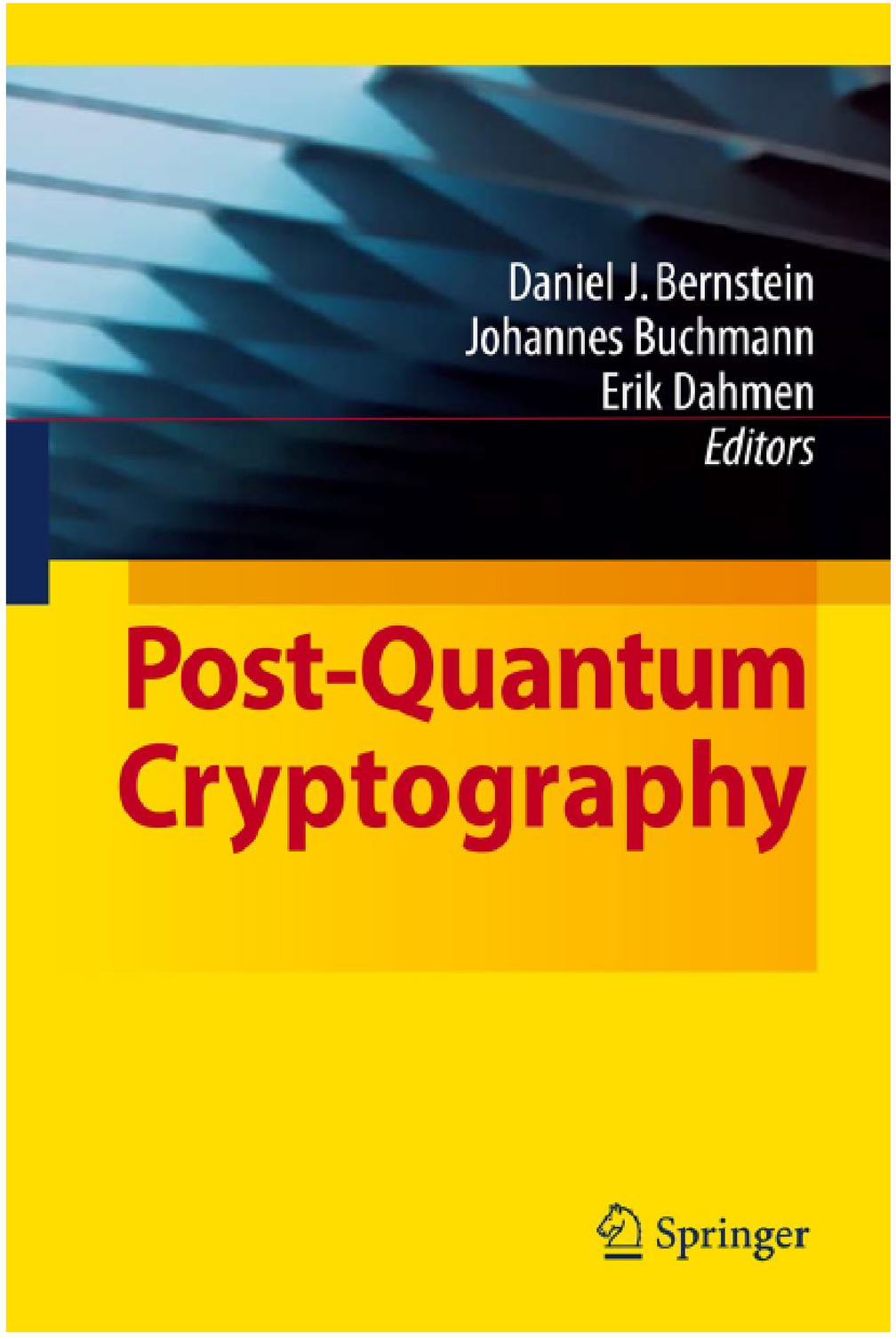

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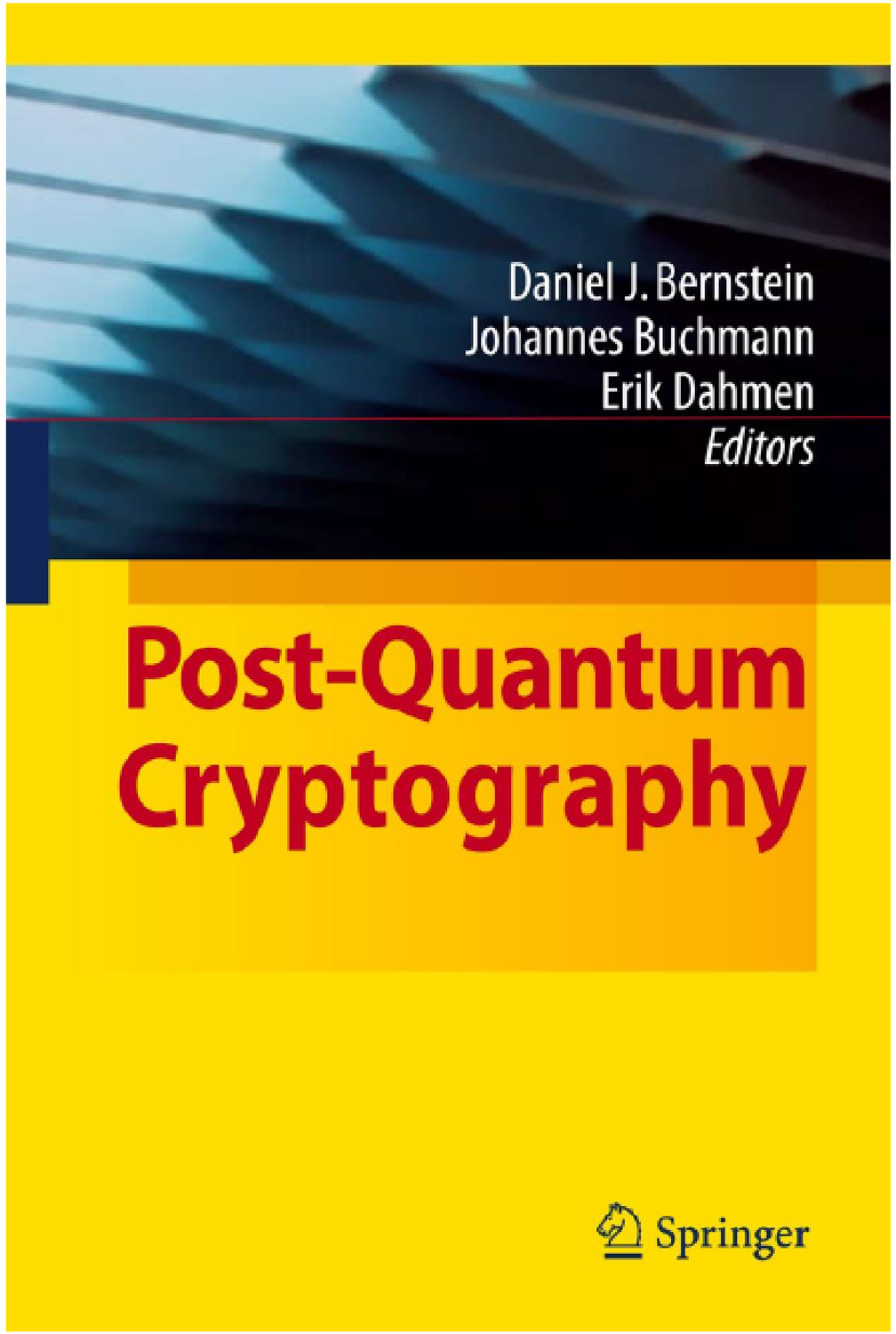

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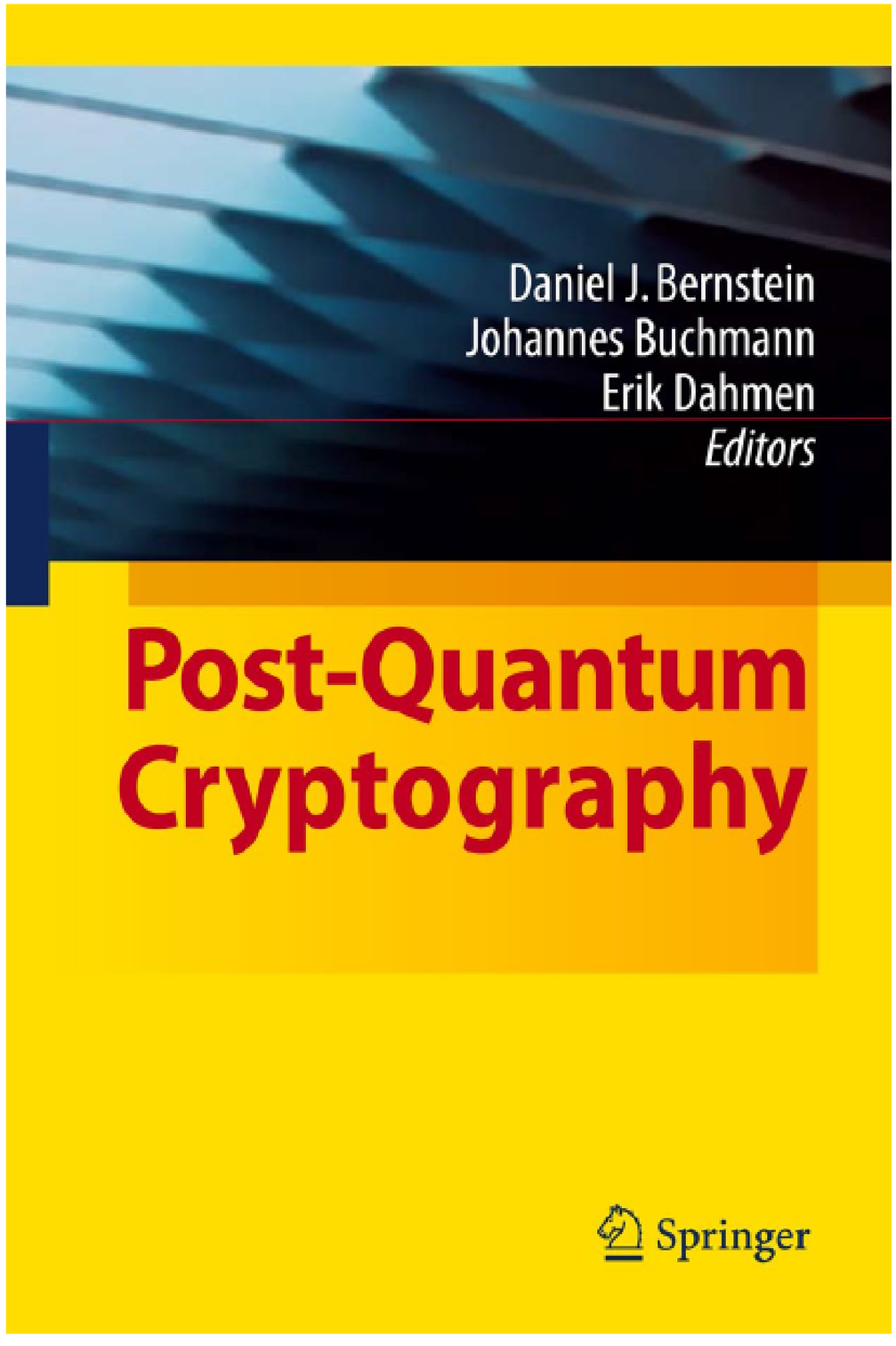

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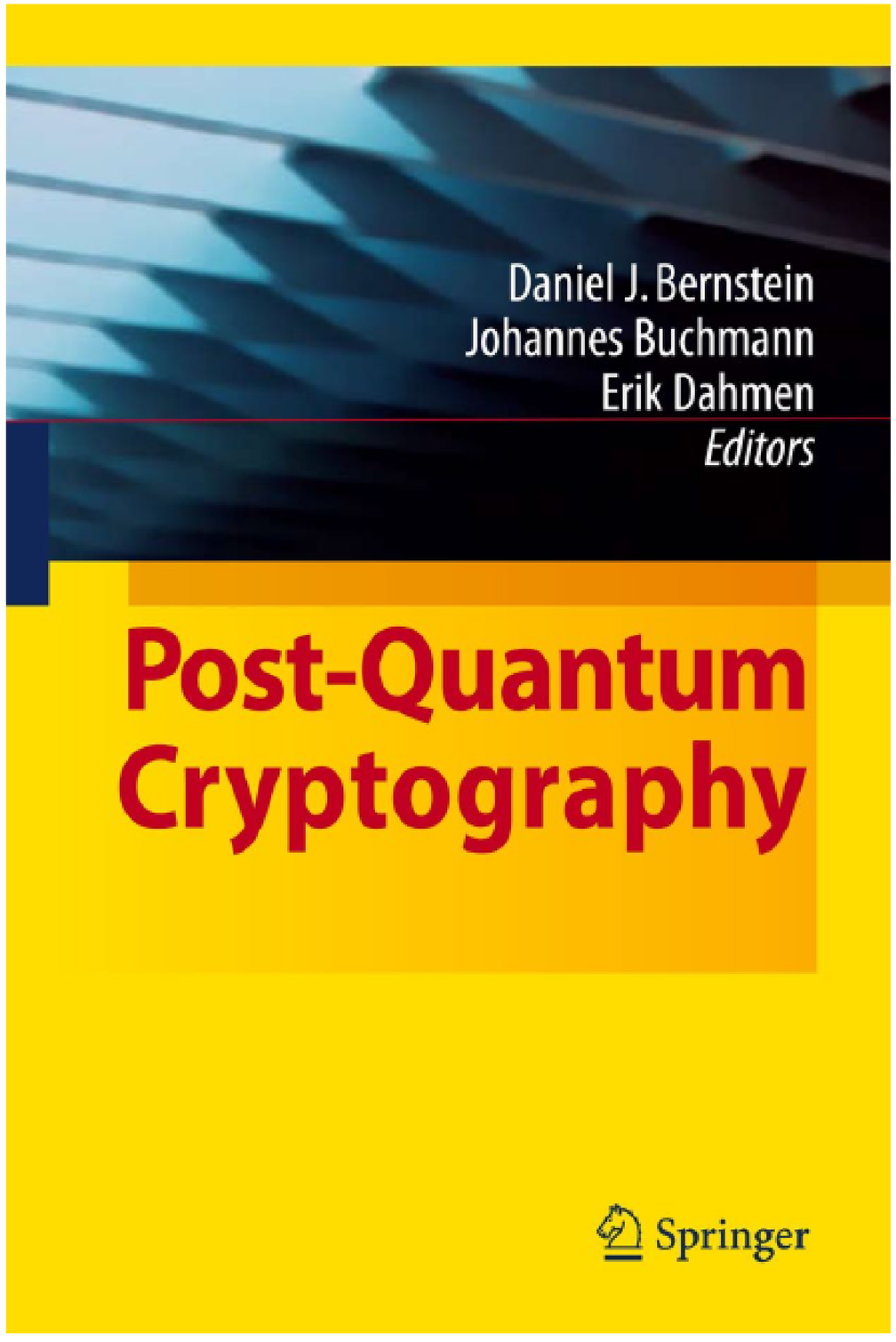

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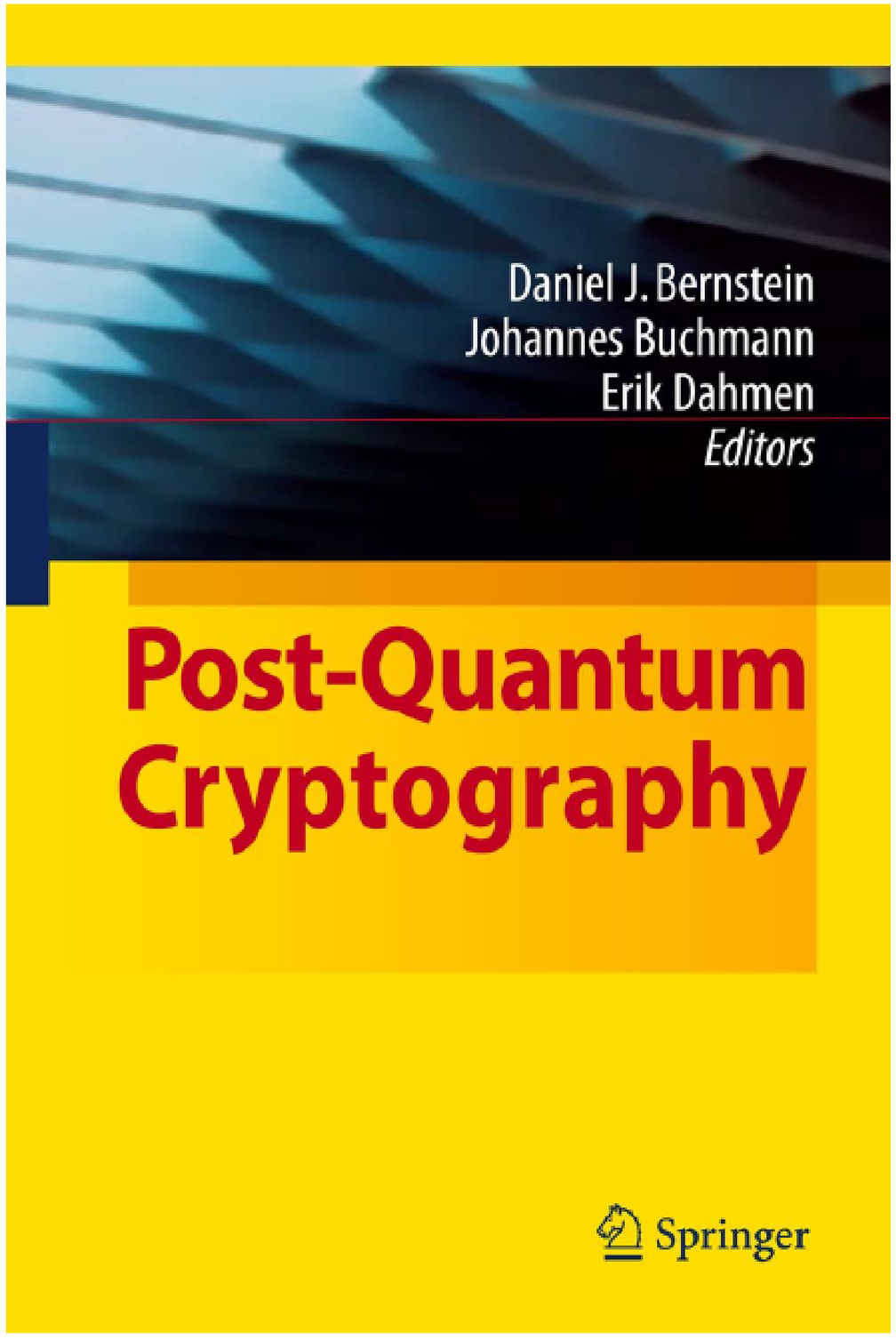

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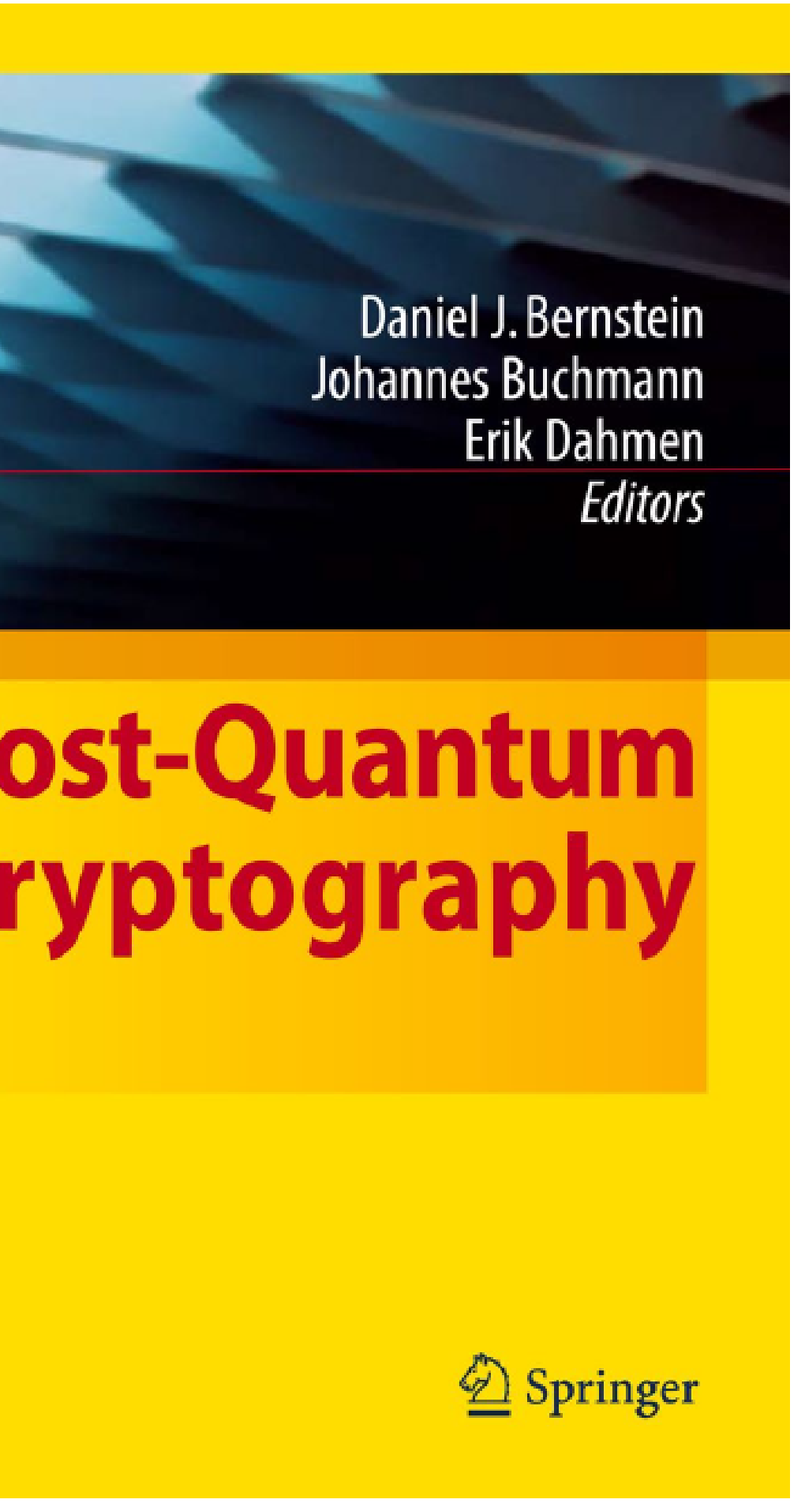

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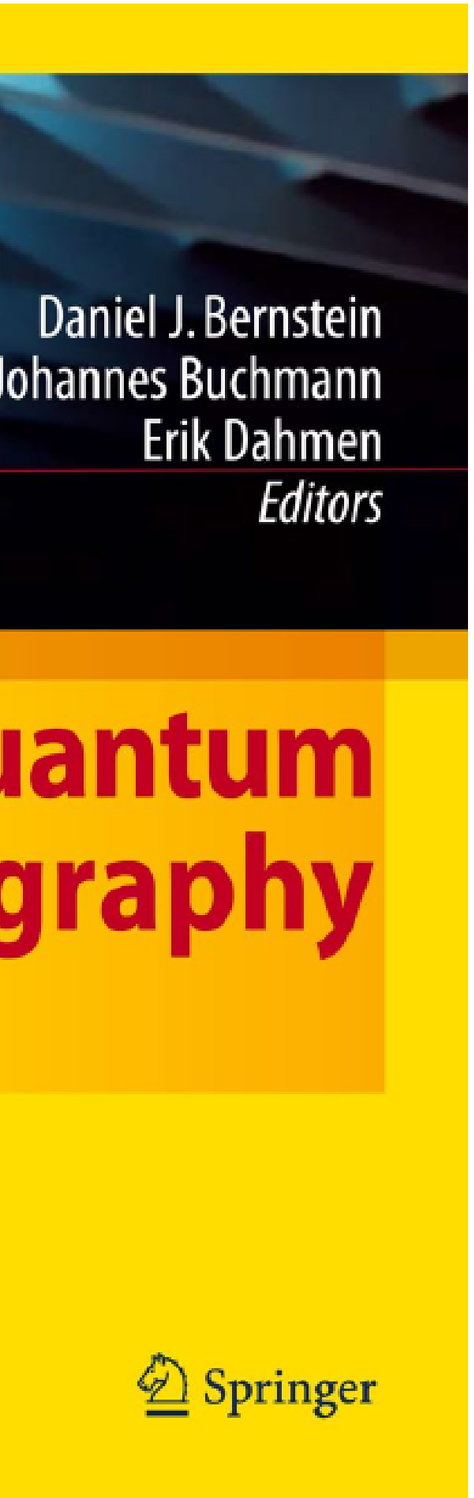

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The McEliece cryptosystem

Receiver's public key: "random"
500×1024 matrix K over $\mathbf{F}_2$.
Specifies linear $\mathbf{F}_2^{1024} \to \mathbf{F}_2^{500}$.

Messages suitable for encryption:
1024-bit strings of weight 50;
i.e., $\{m \in \mathbf{F}_2^{1024} :$
$\#\{i : m_i = 1\} = 50\}$.

Encryption of m is $Km \in \mathbf{F}_2^{500}$.

Can use m as secret AES key
to encrypt much more data.

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Attacker, by linear algebra,
can easily work backwards
from Km to *some* $v \in \mathbf{F}_2^{10}$
such that $Kv = Km$.

i.e. Attacker finds *some*
element $v \in m + \mathrm{Ker}K$.
Note that $\#\mathrm{Ker}K \geq 2^{524}$.

Attacker wants to decode
to find element of $\mathrm{Ker}K$
at distance only 50 from v
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But decoding isn't easy!

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Information-set decoding

Choose random size-500 su
$S \subseteq \{1, 2, 3, \ldots, 1024\}$.

For typical K: Good chanc
that $\mathbf{F}_2^S \hookrightarrow \mathbf{F}_2^{1024} \xrightarrow{K} \mathbf{F}_2^{50}$
is invertible.

Hope $m \in \mathbf{F}_2^S$; chance ≈ 2
Apply inverse map to Km,
revealing m if $m \in \mathbf{F}_2^S$.

If $m \notin \mathbf{F}_2^S$, try again.
$\approx 2^{80}$ operations overall.

Attacker, by linear algebra,
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from Km to *some* $v \in \mathbf{F}_2^{1024}$
such that $Kv = Km$.

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at distance only 50 from v.
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Information-set decoding

Choose random size-500 subset
$S \subseteq \{1, 2, 3, \ldots, 1024\}$.

For typical K: Good chance
that $\mathbf{F}_2^S \hookrightarrow \mathbf{F}_2^{1024} \xrightarrow{K} \mathbf{F}_2^{500}$
is invertible.

Hope $m \in \mathbf{F}_2^S$; chance $\approx 2^{-53}$.
Apply inverse map to Km,
revealing m if $m \in \mathbf{F}_2^S$.

If $m \notin \mathbf{F}_2^S$, try again.
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Information-set decoding

Choose random size-500 subset $S \subseteq \{1, 2, 3, \ldots, 1024\}$.

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Information-set decoding

Choose random size-500 subset $S \subseteq \{1, 2, 3, \ldots, 1024\}$.

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1988 Lee–Brickell idea:
Hope that $m + e \in \mathbf{F}_2^S$
for some weight-2 vector e
Reuse one matrix inversion
for all choices of e.

1989 Stern idea:
Hope that $m + e + e' \in \mathbf{F}$
for low-weight vectors e, e'
Search for collision betwee
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Various improvements:
1988 Lee–Brickell;
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Search for collision between
function of e, function of e'.

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Search for collision between
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Modern McEliece

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Modern McEliece

Easily rescue system by usi
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$(n/2) \times n$ matrix K over **F**
e.g., 1800×3600.

Larger weight: $\approx n/(2 \lg n$
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more reuse, optimization, etc.

Modern McEliece

Easily rescue system by using
a larger public key: "random"
$(n/2) \times n$ matrix K over $\mathbf{F}_2$.
e.g., 1800×3600.

Larger weight: $\approx n/(2 \lg n)$.
e.g. $m \in \mathbf{F}_2^{3600}$ of weight 150.

All known attacks scale badly:
roughly $2^{n/(2 \lg n)}$ operations.
For much more precise analysis
see 2009 Bernstein–Lange–
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ee–Brickell idea:
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Modern McEliece

Easily rescue system by using a larger public key: "random" $(n/2) \times n$ matrix K over $\mathbf{F}_2$. e.g., 1800×3600.

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Receiver secretly generates public key K with a hidden Goppa-code structu that allows fast decoding.

Namely: $K = SHP$ for sec $(n/2) \times (n/2)$ invertible m $(n/2) \times n$ Goppa matrix H $n \times n$ permutation matrix

Detecting this structure seems even more difficult than attacking random K.

Modern McEliece

Easily rescue system by using a larger public key: "random" $(n/2) \times n$ matrix K over $\mathbf{F}_2$. e.g., 1800×3600.

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Goppa

Fix $q \in$
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Detecting this structure
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than attacking random K.

Goppa codes

Fix $q \in \{8, 16, 32$
$t \in \{2, 3, \ldots, \lfloor (q$
$n \in \{t \lg q + 1, t$
e.g. $q = 1024$, t
or $q = 4096$, $t =$

Receiver's matrix
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Receiver secretly generates public key K with a hidden Goppa-code structure that allows fast decoding.

Namely: $K = SHP$ for secret $(n/2) \times (n/2)$ invertible matrix S, $(n/2) \times n$ Goppa matrix H, $n \times n$ permutation matrix P.

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Goppa codes

Fix $q \in \{8, 16, 32, \ldots\}$;
$t \in \{2, 3, \ldots, \lfloor (q-1)/\lg q$
$n \in \{t \lg q + 1, t \lg q + 2, \ldots$
e.g. $q = 1024$, $t = 50$, $n =$
or $q = 4096$, $t = 150$, $n =$

Receiver's matrix H is the parity-check matrix for the classical (genus-0) irreducible length-n degree binary Goppa code defined a monic degree-t irreducibl polynomial $g \in \mathbf{F}_q[x]$ and distinct $a_1, a_2, \ldots, a_n \in \mathbf{F}_q$

Receiver secretly generates
public key K with a
hidden Goppa-code structure
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Namely: $K = SHP$ for secret
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e.g. $q = 1024$, $t = 50$, $n = 1024$.
or $q = 4096$, $t = 150$, $n = 3600$.

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$\times (n/2)$ invertible matrix S,
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e.g. $q = 1024$, $t = 50$, $n = 1024$.
or $q = 4096$, $t = 150$, $n = 3600$.

Receiver's matrix H is
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for the classical (genus-0)
irreducible length-n degree-t
binary Goppa code defined by
a monic degree-t irreducible
polynomial $g \in \mathbf{F}_q[x]$ and
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Monic part of $\epsilon_0 = \alpha_0^2 + x\beta_0^2$
is exactly $\prod_{i:m_i=1}(x - a_i)$.
Factor ϵ_0 and print m.

Why this works:
Define $\epsilon = \prod_{i:m_i=1}(x - a_i)$.
Write ϵ as $\alpha^2 + x\beta^2$ in $\mathbf{F}_q[x]$.
Have $\epsilon'/\epsilon = r$ in $\mathbf{F}_q[x]/g$
so $\beta^2/(\alpha^2 + x\beta^2) = 1/(s^2 + x)$
so $s = \alpha/\beta$ in $\mathbf{F}_q[x]/g$;
i.e., $(\alpha, \beta) \in L$.
Volume of L forces
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Apply to the Goppa situation: analogous algorithm finds all divisors of $\prod_i (x - a_i)$ that are linear combinations of ϵ_0, ϵ_1 with small coprime coefficients.

Compared to Patterson, pushes allowable weight of m up to $\approx t + t^2/n$.

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Many other current topics in code-based cryptography

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Encryption time $b^{\lg 3+o(1)}$.
Decryption time $b^{\lg 3+o(1)}$.

2009 Bernstein: easy tweak
to Misoczki–Barreto algorithms,
reducing time to $b^{1+o(1)}$.

Last slide: advertisements

2009.09.09–10, Lausanne:
Special-purpose Hardware
Attacking Cryptographic S

2009.10.12–13, Berlin: Sof
Performance Enhancement
Encryption and Decryption
and Cryptographic Compile
Submission deadline 2009.

2010.05.25–28, Darmstadt
Third International Worksh
on Post-Quantum Cryptog
Submission deadline 2009.

Many other current topics
in code-based cryptography.

e.g. 2009 Misoczki–Barreto:
Hide quasi-dyadic Goppa code
as quasi-dyadic public key.
Key length only $b^{1+o(1)}$.
Encryption time $b^{\lg 3+o(1)}$.
Decryption time $b^{\lg 3+o(1)}$.

2009 Bernstein: easy tweak
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reducing time to $b^{1+o(1)}$.

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2009.09.09–10, Lausanne:
Special-purpose Hardware for
Attacking Cryptographic Systems.

2009.10.12–13, Berlin: Software
Performance Enhancement for
Encryption and Decryption
and Cryptographic Compilers.
Submission deadline 2009.08.30.

2010.05.25–28, Darmstadt: The
Third International Workshop
on Post-Quantum Cryptography.
Submission deadline 2009.11.15.