

STAT 217
Assignment #1

Note: answers may vary slightly due to rounding and whether or not you use the computer or the tables.

MINITAB INSTRUCTIONS

CALC ⇒ Probability Distributions ⇒ normal.

Finding the area above and below a Z-value under the Standard Normal Curve

For a given Z-value, you are required to find a probability. In the dialog box, which corresponds to the Normal distribution, you have three choices:

- Probability**
- Cumulative probability**
- Inverse cumulative probability**

Click **Cumulative probability**. This will calculate the cumulative probability associated with a specific Z-value (or the area under the Standard Normal Curve to the **left** of a specific Z-value.)

The middle of the dialog box has 2 options:

- Mean**
- Standard Deviation**

The default values for the **Mean** and **Standard Deviation** are **0** and **1**, respectively. There is no need to change these values, so just leave these as is.

In the bottom portion of the dialog box, select **Input constant**. It is in this box that you enter a specific Z-value. Once you have completed this, either press return or “click” on **OK**. In the upper portion of your screen, or the command module, MINITAB will return the area to the **left** of the Z-value you have entered. Note that when this routine is employed, the probability returned is **ALWAYS THE AREA TO THE LEFT OF** the Z-value entered above or $P(Z < z)$

For practice, try question 1 using this routine.

1. Given that Z is a standard normal random variable, compute the following probabilities:

- (a) $P(-0.72 \leq Z \leq 0)$ (.2642)
- (b) $P(-0.35 \leq Z \leq 0.35)$ (.2736)
- (c) $P(0.22 \leq Z \leq 1.87)$ (.3822)
- (d) $P(Z \leq -1.02)$ (.1539)
- (e) $P(Z \geq -0.88)$ (.8106)
- (f) $P(Z \geq 1.38)$ (.0838)
- (g) $P(-0.34 \leq Z \leq 2.33)$ (.6232)

Finding a Z-value for a given area (or probability under the Standard Normal Curve)

For a given probability, you are required to find a Z-value that corresponds to this probability. This requires the use of the **Inverse cumulative probability** routine in the dialog box employed above.

“Click” on the circle which corresponds to **Inverse cumulative probability** and just as was done previously, do not touch the box labeled **Mean** and **Standard Deviation**.

This routine needs an area, and will subsequently find the Z-value which matches up with the area entered. Just as was done above, move your mouse down to the bottom portion of the dialog box and “click” on the circle which corresponds to **Input constant**. Previously you entered a Z-value here. But now you want to

find a Z-value for a given area, or probability. So the number you will enter in the **Input constant** box is a probability, or an area to the left of the Z-value in question. Once you have entered the correct probability, either press return or “click” on **OK**. MINITAB will return a Z-value in the command module on the upper portion of your screen.

A good rule-of-thumb in these types of problems is to draw your standard normal curve and piece together the areas given. The Z-value will be given when you specify the area to the left of that value. Practice this routine on question 2.

2. Given that Z is a standard normal random variable, determine Z_o if it is known that:

- (a) $P(-Z_o \leq Z \leq Z_o) = 0.90$ (1.645)
- (b) $P(-Z_o \leq Z \leq Z_o) = 0.10$ (.1257)
- (c) $P(Z \geq Z_o) = 0.20$ (.842)
- (d) $P(-1.66 \leq Z \leq Z_o) = 0.25$ (-.529)
- (e) $P(Z \leq Z_o) = 0.40$ (-.253)
- (f) $P(Z_o \leq Z \leq 1.80) = 0.20$ (.720)

3. Every year around Halloween in the bustling metropolis of Red Deer, many street signs are defaced. The average repair cost per sign is \$68.00 and the standard deviation is \$12.40.

- (a) What is the probability that a defaced sign’s repair cost will exceed \$85.00? (.0852)
- (b) In one particular year, 36 signs were vandalized. What is the probability that the average repair cost of these signs is between \$67.00 and \$70.00? (0.5184)

4. According to a article in the January 1991 issue of *Health* magazine, the cost of a root canal ranges from \$200 to \$700. Suppose the mean cost for root-canal therapy is \$450 and the standard deviation is \$125. If a sample of 100 dentists from across the country was taken, what is the probability that the mean cost of a root-canal will fall between \$425 and \$475 (.9554)

5. The time a recreational skier takes to go down a downhill course has a normal distribution with a mean of 12.3 minutes and a standard deviation of 0.4 minutes.

- (a) What is the probability that the skier will take between 12.1 and 12.6 minutes to complete a run on the course? (.4649)
- (b) What is the maximum time (in minutes) the skier must have for the time to be classified “among the fastest 10% of his times”? (11.7874)
- (c) The times for a random sample of 4 of the skier’s runs is considered. What is the probability that the average time for this sample is more than 12.75 minutes? (0.0122)

6. The time it takes Bob to get from home to his office follows a normal distribution. The probability that it takes him less than 3 minutes is 0.345. The probability that it takes him more than 10 minutes is 0.01. Find the average time and variance (μ and σ^2) of this normal distribution. (4.0246, 6.5978)

7. “Your Eyes”, a daily eye ware retail store, serves an average of 14.3 customer per day. Assume that the distribution of the number of customer served per day has a standard deviation of 5.9. What is the probability that the average number of customers served per day will be:

- (a) at least 15, based on a random sample of 50 days? (.2008)
- (b) less than 14, based on a random sample of 50 days? (0.3598)

8. Electrical connectors last on average 18.2 months with a standard deviation of 1.7 months. Assume that the life of the connectors is normally distributed.

- (a) The manufacturer agrees to replace, free of charge, any connectors that fail within 17 months of installation. What percentage of the connectors can he expect to have replaced free of charge? (.2404)
 - (b) The manufacturer does not want to have to replace more than 2.5% of the connectors free of charge. What should he set the life at for free replacement? (14.868)
 - (c) This question will not be on the quiz, but could be asked as a bonus question. What is the probability that the total lifetime of 24 connectors will exceed 38 years? ($z = 2.3054$, prob. ~ 0.010)
9. A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of the sample will
- (a) exceed 52.9? (0.0388)
 - (b) fall between 50.5 and 52.3? (0.7104)
 - (c) be less than 50.6? (0.1734)

The directions for the t distribution for MINITAB is the same as the directions for the standard normal. The only difference is that you have to also plug in degrees of freedom.

Note: If you want to calculate the mean and standard deviation of a data set,

1. input all the data into one column.
2. Click on the header Calc>Column Statistics.
3. Click on the statistic that you are interested in (mean, st.dev etc)
4. Type the column in which the data is in, in the input variable box (or click on the input variable box and then double click on the column where the data is located.
5. Hit enter or click on OK

Note: You should familiarize yourself with some of the other functions of MINITAB. Check them out. You may find some time saving techniques.

Confidence intervals

1. In developing patient appointment schedules, a medical center desires to estimate the mean time a staff member spends with each patient. How large a sample should be taken if the precision of the estimate is to be ± 2 minutes at a 95% level of confidence? How large a sample is needed for a 99% level of confidence? Use a planning value for the population standard deviation of 8 minutes. [62, 107]
2. A simple random sample of five people provided the following data on ages: 21, 25, 20, 18, and 21. Develop a 95% confidence interval for the mean age of the population being sampled. State any assumptions you must make in your method. [17.8349, 24.1651]
3. The time (in minutes) taken by a biological cell to divide into two cells has a normal distribution. From past experience, the standard deviation can be assumed to be 3.5 minutes. When 16 cells were observed, the mean time taken by them to divide was 31.2 minutes. Estimate the true mean time for a cell division using a 98 percent confidence interval. [29.1645, 33.2355]
4. Based on a random sample of 100 cows of a certain breed, a confidence interval for estimating the true mean yield of milk is given by $41.6 < \mu < 44.0$. If the yield of milk of a cow may be assumed to be normally distributed with a standard deviation of 5, what was the level of confidence used? [98.36%]
5. When 16 cigarettes of a particular brand were tested in a laboratory for the amount of tar content, it was found that their mean content was 18.3 milligrams with a standard deviation of 1.8 milligrams. Set a 90 percent confidence interval for the mean tar content in cigarettes of this brand. (Assume that the amount of tar in a cigarette is normally distributed.) [17.5111, 19.0889]
6. In 10 half-hour programs on a TV channel, Mary found that the number of minutes devoted to commercials were 6, 5, 5, 7, 5, 4, 6, 7, 5, and 5. Set a 95% confidence interval for the true mean time devoted to commercials during a half-hour program. Assume that the amount of time devoted to commercials is normally distributed. [4.8049, 6.1951]

7. A random sample of 16 servings of canned pineapple has a mean carbohydrate content of 49 grams. If it can be assumed that population is normally distributed with a variance of 4 grams, find a 98 percent confidence interval for the true mean carbohydrate content of a serving. [47.835, 50.165]
8. An archaeologist found that the mean cranial width of 17 skulls was 5.3 inches with a standard deviation of 0.5 inches. Using a 90% confidence level, set a confidence interval for the true mean cranial width. Assume that the cranial width is normally distributed. [5.0883, 5.5117]
9. It is suspected that a substance called actin is linked to various movement phenomena of non-muscle cells. In a laboratory experiment when eight fertilized eggs were incubated for 14 days the following amounts (mg) of total brain actin were obtained: 1.2, 1.4, 1.5, 1.2, 1.4, 1.7, 1.5, 1.7. Assuming that brain-actin amount after 14 days of incubation is normally distributed,
 - (a) Find a 95 percent confidence interval for the true mean brain-actin amount. [1.2890, 1.6111]
 - (b) How can we decrease/increase the error? Assume that the variability does not change from the data given above.
10. An economist wants to estimate the mean income for the first year of work for a college graduate who has had the profound wisdom to take a statistics course. How many such incomes must be found if we want to be 95% confident that the sample mean is within \$500 of the true population mean? Assume that a previous study has revealed that for such incomes, $\sigma = 6250$. [601]
11. If we want to estimate the mean weight of plastic discarded by households in one week, how many households must we randomly select if we want to be 99% confident that the sample mean is within 0.250 lb of the true population means when preliminary results show that the standard deviation is 1.067? [121]
12. Wawanesa Mutual Insurance Company wants to estimate the percentage of drivers who change tapes or CDs while driving. A random sample of 850 drivers results in 544 who change tapes or CDs while driving.
 - (a) Find the point estimate of the percentage of all drivers who change tapes or CDs while driving. [64.0%]
 - (b) Find a 90% interval estimate of the percentage of all drivers who change tapes or CDs while driving. [61.29% < p < 66.71%]
13. In a study of store checkout scanners, 1234 items were checked and 20 of them were found to be overcharges.
 - (a) Using the sample data, a confidence interval for the proportion of all such scanned items that are overcharges was found to be from 0.00915 to 0.02325. What was the level of confidence that was used? [~95% level of confidence]
 - (b) Find the sample size necessary to estimate the proportion of scanned items that are overcharges. Assume that you want 99% confidence that the estimate is in error by no more than 0.005.
 - (i) Use the sample data as a pilot study [4228]
 - (ii) Assume, instead, that we do not have prior information on which to estimate the value of \hat{p} . [66307]
14. A hotel chain gives an aptitude test to job applicants and considers a multiple-choice test question to be easy if at least 80% of the responses are correct. A random sample of 6503 responses to one particular question includes 84% correct responses. Construct the 99% confidence interval for the true percentage of correct responses. Is it likely that the question is really easy? [82.83% < p < 85.17%, yes]