MATHEMATICS 271 L03 WINTER 2002
QUIZ 1

[8] **1**. For each of the following questions, give a brief explanation on how you got the answer.
(a) How many integers are there in the range from 100 to 999 in which only the digits 1,2,3,4,5,6 and 7 are used?
(b) Among the integers in (a), how many have the property that all the digits used are distinct?
(c) Among the integers in (a), how many are even?

[7] **2**. Let $\mathcal{P}$ be the statement: "If x is rational and $x+y$ is irrational then y is irrational." For each of the following question, you can use the fact that $\sqrt{2}$ is irrational. However, if you use any other irrational numbers, you must prove that it is indeed irrational.

(a) Is $\mathcal{P}$ true for all real numbers x and y? Prove your answer.
(b) Write the converse of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true for all real numbers x and y? Prove your answer.

QUIZ 2

[7] **1**. For each of the following statements, determine if the statement is true or false and prove your answer.
(a) For all sets A, B and C, $A-(B\cup C)\subseteq(A-B)\cup C$.
(b) For all sets A, B and C, $(A-B)\cup C\subseteq A-(B\cup C)$.

[8] **2**. Let $\mathcal{P}$ be the statement: "if $A=B-C$ then $B=A\cup C$."
(a) Is $\mathcal{P}$ true for all sets A, B and C? Prove your answer.
(b) Is the converse of $\mathcal{P}$ true for all sets A, B and C? Prove your answer.

QUIZ 3

[7] **1**. Prove by induction that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n\,(n+1)}=1-\frac{1}{n+1} \text{ for all integers } n\geq 1.$$

[8] **2**. Let $\mathcal{R}$ be a relation on $\mathbf{Z}$ ($\mathbf{Z}$ is the set of all integers) defined by:
for all $x,y\in\mathbf{Z}$, $\quad x\mathcal{R}y$ if and only if $2x+y$ is divisible by 3.
(a) Prove that is an equivalence relation on $\mathbf{Z}$.
(b) Find three positive and three negative elements of the equivalence class $[-2]$.

QUIZ 4

[8] **1**. Let $f : \mathbf{Z} \to \mathbf{Z}$ be a function ($\mathbf{Z}$ is the set of all integers) defined by:
$f(x) = 4x^2 + x - 3$ for all $x \in \mathbf{Z}$.
(a) Is f onto? Prove your answer.
(b) Is f one-to-one? Prove your answer.

[7] **2**. Let $A \subseteq \mathbf{Z}$ where is the set of all integers. Let $\mathcal{R}$ be a relation on $A \times A$ defined by:
for all $a, b, c, d \in A$, $(a, b) \mathcal{R} (c, d)$ if and only if $a \leq c$ and $b \geq d$.
(a) Prove that $\mathcal{R}$ is a partial order on $A \times A$.
(b) Draw the Hasse diagram of the poset $(A \times A, \mathcal{R})$ when $A = \{1, 2, 3\}$.
(c) Find two linear extensions of $\mathcal{R}$ when $A = \{1, 2, 3\}$.

QUIZ 5

[6] **1**. A bag contains ten identically wrapped boxes which contains different amounts of money from \$1.00 to \$10.00. Suppose that Alice picks one of the ten boxes at random and then Bob picks at random one of the remaining boxes, What is the probability that Alice gets more money than Bob does? Is there an advantage of going first? **Simplify your answer.**

[9] **2**. Let $X = \{1, 2, 3, 4, 5, 6\}$. Let $\mathcal{F}$ be the sets of all functions from X to X. Choose an element f of $\mathcal{F}$ and assume that all elements of F are equally likely. Let A be the event that f is one-to-one anf let B be the event that $f(1)$ is a prime.
(a) Find $P(A)$.
(b) Find $P(B)$.
(c) Are A and B independents? Explain.

QUIZ 6

[7] **1**. In this question, the three parts are related.
(a) Find $\gcd(844, 375)$.
(b) Is 375 invertible in Z_{844}? Explain. If 375 is invertible in Z_{844}, find the inverse (reciprocal) of 375 in Z_{844}.
(c) In Z_{844}, solve the equation $375 \otimes x = 2$.

[4] **2**. Prove that if a, n are integers with $n > 0$ and a and n are relatively prime then there is an integer b so that $ab \equiv 1 \pmod{n}$.

[4] **3**. Let a, b, c be integers. Prove that if a and b are relatively prime and that $a \mid c$ and $b \mid c$ then $ab \mid c$.