

MATHEMATICS 271 L01 FALL 2003
QUIZ 6 SOLUTION

1. (a) Give the definition of an r -regular graph.

Solution: An r -regular graph is a graph in which every vertex has degree r .

(b) Find all 3-regular graphs with 9 vertices. Explain how you get the answer.

Solution: There are no 3-regular graphs with 9 vertices. Suppose that there is a 3-regular graph with 9 vertices, then the sum of the degrees is $9 \times 3 = 27$, which contradicts the fact that in any graph, the sum of the degrees must be even, for being twice the number of edges.

2. Recall that $\alpha(G)$ the independence number of G . Prove or disprove the statement:

“If G is a subgraph of H then $\alpha(G) \leq \alpha(H)$.”

Solution: This statement is false. For example, when H is the complete graph with vertices 1 and 2, and G is the graph with vertices 1 and 2, and $E(G) = \emptyset$, we have $\alpha(G) = 2 > \alpha(H) = 1$.

3. Let K_n be a complete graph with n vertices where $n \geq 2$.

(a) How many edges does K_n have? Explain.

Solution: K_n has $\binom{n}{2} = \frac{n(n-1)}{2}$ edges, because every two vertices of K_n determine an edge.

(b) How many spanning subgraphs does K_n have? Explain.

Solution: K_n has $2^{\binom{n}{2}}$ spanning subgraphs. This is because if G a spanning subgraph of K_n then G must have all the vertices of K_n and $E(G)$ is a subset of $E(K_n)$, and there are $2^{\binom{n}{2}}$ subsets of $E(K_n)$ because $E(K_n)$ has $\binom{n}{2}$ elements as shown in (a).