MATHEMATICS 271 L01 FALL 2003
QUIZ 2 SOLUTION

1. Prove or disprove each of the following statement:
(a) For all sets A, B and C, if $A - B \subseteq A - C$ then $C \subseteq B$.
Solution:

This statement is false. For example, when $A = B = \emptyset$ and $C = \{1\}$, we have $A - B = \emptyset - B = \emptyset \subseteq A - C$, but $C = \{1\} \not\subseteq \emptyset = B$.

(b) For all sets A, B and C, if $C \subseteq B$ then $A - B \subseteq A - C$.
Solution:

This statement is true and here is a proof. Let A, B and C be sets so that $C \subseteq B$. We show that $A - B \subseteq A - C$. Let $x \in A - B$; that is, $x \in A$ and $x \notin B$. We show that $x \notin C$ by a contradiction proof. Suppose that $x \in C$. Then from $x \in C$ and $C \subseteq B$ we have $x \in B$ which contradicts the fact that $x \notin B$. Thus, $x \notin C$. Now, from $x \in A$ and $x \notin C$, we get $x \in A - C$.

Thus, $A - B \subseteq A - C$.

2. Four-digit positive integers are formed by using the digits 1,2,3,4,5,6 and 7. For each of the following questions, give a brief explanation on how you got the answer.
(a) How many such integers are there so that none of its digits is repeated?
Solution: The answer to this question is $7 \times 6 \times 5 \times 4$. This is because we have 7 choices for the first digit, 6 choices for the second digit, 5 choices for the third digit and 4 choices for the last digit.
(b) How many such integers are there so that the digit 1 appears at least once (in this case the digits may be repeated)?
Solution: The answer to this question is $7^4 - 6^4$.

Let S be the set of all four-digit positive integers formed by using the digits 1,2,3,4,5,6 and 7. Then $|S| = 7^4$ because there are 7 choices for each of the four digits. Now, let B be the set of integers in S in which the digit 1 appears at least once. Then $S - B$ is the set of integers in S in which the digit 1. Then $|S - B| = 6^4$ because there are 6 choices for each of the four digits. The answer to this question is $|B| = |S| - |S - B| = 7^6 - 6^4$.
(c) How many such integers are there so that none of its digits is repeated or the digit 1 appears at least once (or both)?
Solution: The answer to this question is $7^4 - 6^4 + 3 \times 6 \times 5 \times 4$.

Let A be the set of integers in S so that none of its digits is repeated. We know by (a) that $|A| = 7 \times 6 \times 5 \times 4$. With B as in (b), we see that $A \cap B$ is the set of integers in S so that none of its digits is repeated and the digit 1 appears at least once. Then $|A \cap B| = 4 \times 6 \times 5 \times 4$ because there are 4 choices of where to put the digit 1, and there are $6 \times 5 \times 4$ ways of arranging three of the 6 remaining digits. The answer to this question is $|A \cup B| = |A| + |B| - |A \cap B| = 7 \times 6 \times 5 \times 4 + 7^6 - 6^4 - 4 \times 6 \times 5 \times 4 = 7^6 - 6^4 + 3 \times 6 \times 5 \times 4$.