MATHEMATICS 271 L01 FALL 2003
QUIZ 1 SOLUTION

1. Write the *negation* (in good English) of each of the following statements. The answer " It is not the case that ..." is **not** acceptable.
(a) For all real numbers x and y, if x is rational and y is irrational then $x+y$ is irrational and xy is irrational.
(b) There exists a positive number x so that for all positive numbers y, $x \leq y$.
(c) For all positive numbers y, there exists a positive number x so that $x \leq y$.
Solution:
(a) There are real numbers x and y so that x is rational and y is irrational, but $x+y$ is rational or xy is rational.
(b) For all positive numbers x, there is a positive numbers y so that $x > y$.
(c) There is a positive number y so that for all positive numbers x, $x > y$.

2. Let $\mathcal{P}$ be the statement:

$$\text{If } d \mid a \text{ and } d \mid b \text{ then } d \mid (a+b) \text{ and } d \mid (a-b).$$

(a) Write the *converse* of $\mathcal{P}$.
(b) Write the *contrapositive* of $\mathcal{P}$.
(c) Write the *negation* of $\mathcal{P}$.
Solution:
(a) If $d \mid (a+b)$ and $d \mid (a-b)$ then $d \mid a$ and $d \mid b$.
(b) If $d \nmid (a+b)$ or $d \nmid (a-b)$ then $d \nmid a$ or $d \nmid b$.
(c) $d \mid a$ and $d \mid b$, and $d \nmid (a+b)$ or $d \nmid (a-b)$.

3. Let $\mathcal{Q}$ be the statement:

$$\text{If } n \text{ is odd then } n^2 + 2n \text{ is odd.}$$

(a) Is $\mathcal{Q}$ true for all integers n? Prove your answer.
(b) Write the *converse* of $\mathcal{Q}$. Is the *converse* of $\mathcal{Q}$ true for all integers n? Prove your answer.
Solution:
(a) $\mathcal{Q}$ is true for all integers n.
Proof: Let n be an odd integer. Then $n = 2k+1$ for some integer k, and so $n^2 + 2n = (2k+1)^2 + 2n = 4k^2 + 4k + 1 + 2n = 2\left(2k^2 + 2k + n\right) + 1$, which, together with the fact that $2k^2 + 2k + n$ is an integer (for k and n are integers), imply that $n^2 + 2n$ is odd.
(b) The *converse* of $\mathcal{Q}$ is: "If $n^2 + 2n$ is odd then n is odd. The *converse* of $\mathcal{Q}$ is true for all integers n.
Proof: Let n be an integer and suppose that $n^2 + 2n$ is odd. We prove that n is odd by a contradiction proof. Suppose that n is not odd, that is, n is even. Then $n = 2m$ for some integer m, and so $n^2 + 2n = (2m)^2 + 2n = 4m^2 + 2n = 2\left(2m^2 + n\right)$, which, together with the fact that $2m^2 + n$ is an integer (for m and n are integers), imply that $n^2 + 2n$ is even. This contradicts the assumption that $n^2 + 2n$ is odd. Thus, n is odd.