

MATHEMATICS 271 L01 FALL 2003
ASSIGNMENT 4

Due at NOON on Friday, November 14. Your assignment must be handed in at the beginning of the lab on November 14. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

Marked assignments will be returned during the lab on Friday, November 21.

1. Let a, b, c be integers.

- (a) Prove that if $c \mid a$ and $c \mid b$ then $c \mid (xa + yb)$ for any integers x and y .
- (b) Prove that if a and b are relatively prime and, $a \mid c$ and $b \mid c$ then $(ab) \mid c$.
- (c) Prove that if a and b are relatively prime and $a \mid bc$ then $a \mid c$.

2. Let $A = \{1, 2, 3, 4\}$ and let \mathcal{F} be the set of all functions from A to A . Choose an elements f of \mathcal{F} and assume that all elements of \mathcal{F} are equally likely.

- (a) What is the probability that f is one-to-one?
- (b) What is the probability that $f(1) \leq f(2)$?
- (c) Given that f is one-to-one, what is the probability that $f(1) \leq f(2)$?
- (d) Are the event that f is one-to-one and the event that $f(1) \leq f(2)$ independent? Explain.

3. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove or disprove each of the following.

- (a) If f and g are one-to-one then $g \circ f$ is also one-to-one.
- (b) If $g \circ f$ is one-to-one then f must be one-to-one.
- (c) If $g \circ f$ is one-to-one then g must be one-to-one.
- (d) If $g \circ f$ is one-to-one and f is onto then g is one-to-one.