MATHEMATICS 271 L01 FALL 2003
ASSIGNMENT 3

Due at NOON on Friday, October 24. Your assignment must be handed in at the beginning of the lab on October 24. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.
Marked assignments will be returned during the lab on Friday, October 31.

1. Prove the following statements by induction on n.
(a) $3^n + 1$ is divisible by 2 for all integers $n \geq 1$.
(b) $5^{n+1} + 2 \times 3^n + 1$ is divisible by 8 for all integers $n \geq 1$.

2. Let $n \geq k \geq 1$ be natural numbers.
(a) Prove that $\binom{n}{0} + \binom{n}{1} + ... + \binom{n}{n} = 2^n$ using the Binomial Theorem (Theorem 14.8).
(b) Prove that $\binom{n}{0} + \binom{n}{1} + ... + \binom{n}{n} = 2^n$ by a combinatorial proof.
(c) Prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$.
(d) Prove that $1\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + ... + n\binom{n}{n} = n2^{n-1}$.

3. For positive integers n, let S_n be the number of ways to display flags on an n foot tall flagpole using red flags (which are 1 foot tall), blue flags (which are 2 feet tall) and green flags (which are 2 feet tall).
(a) Find S_1 and S_2. Explain how you get the answers.
(b) Show that for $n \geq 3$, $S_n = 2S_{n-2} + S_{n-1}$.
(c) Prove by induction on n that $S_n = \frac{2}{3}2^n + \frac{1}{3}(-1)^n$.