MATHEMATICS 271 L01 FALL 2003
ASSIGNMENT 5

Due at 11:00 a.m. on Wednesday, December 3. Your assignment must be handed in at the beginning of the lecture on December 3. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.
Marked assignments will be returned during the lecture on Monday, December 8.

1. In this question, a, b, d are positive integers. Prove or disprove each of the following statements.

(a) $d \mid a$ and $d \mid b$ if and only if $d \mid (xa + yb)$ for all integers x and y.

(b) $\gcd(a, b) \leq \gcd(xa + yb, ma + nb)$ for all integers x, y, m and n.

(c) For all positive integers a and b, $\gcd(a, b) = \gcd(2a + 3b, 2a + b)$.

(d) For all positive integers a and b, $\gcd(a, b) = \gcd(2a + 3b, a + 2b)$.

2. Let $n \geq 2$ be an integer. Let G_n be the graph where $V(G_n) = \{2, 3, 4, ..., n\}$ and a vertex a is connected to vertex b by an edge if and only if a and b are relatively prime. For each of the following questions from part (b) to part (e), an explanation is required to support your answer.

(a) Draw the graphs G_5 and G_6.

(b) Find the degree of the vertex 2 in G_n.

(c) Find the degree of the vertex 3 in G_n for $n \geq 3$.

(d) Is it true that G_n has a Hamiltonian path for all integers $n \geq 2$? (A Hamiltonian path of a graph G is a path that contains all the vertices of G.)

(e) Find all values of n so that G_n has a Hamiltonian cycle? (A Hamiltonian cycle of a graph G is a cycle that contains all the vertices of G.)

3. (This is exercise #13 on page 376 of the text book) Let n and k be integers with $1 \leq k < n$. Let $G_{n,k}$ be the graph with $V(G_{n,k}) = \{0, 1, 2, 3, ..., n-1\}$ and $E(G_{n,k}) = \{ab : a - b \equiv \pm k \pmod{n}\}$.

(a) Draw the graphs $G_{6,1}$, $G_{6,2}$ and $G_{6,3}$.

(a) Find necessary and sufficient conditions on n and k so that G is connected. Prove your answer.

(b) Find a formula involving n and k for the number of connected components of G. Explain how you get the formula.