A SHORT TABLE OF INDEFINITE INTEGRALS

I. Basic Functions

1. $\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\displaystyle\int a^x\,dx = \frac{1}{\ln a}a^x + C$
4. $\displaystyle\int \ln x\;dx = x\ln x - x + C, \quad x > 0$
5. $\displaystyle\int \sin x\;dx = -\cos x + C$
6. $\displaystyle\int \cos x\;dx = \sin x + C$

7(a) $\displaystyle\int \tan x\;dx = -\ln|\cos x| + C$

7(b) $\displaystyle\int \cot x\;dx = \ln|\sin x| + C$

7(c) $\displaystyle\int \sec^2 x\;dx = \tan x + C$

7(d) $\displaystyle\int \csc^2 x\;dx = -\cot x + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\displaystyle\int e^{ax}\sin(bx)dx = \frac{1}{a^2+b^2}e^{ax}\left[a\sin(bx) - b\cos(bx)\right] + C$
9. $\displaystyle\int e^{ax}\cos(bx)dx = \frac{1}{a^2+b^2}e^{ax}\left[a\cos(bx) + b\sin(bx)\right] + C$
10. $\displaystyle\int \sin(ax)\sin(bx)dx = \frac{1}{b^2-a^2}\left[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)\right] + C, \quad a \neq b$
11. $\displaystyle\int \cos(ax)\cos(bx)dx = \frac{1}{b^2-a^2}\left[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)\right] + C, \quad a \neq b$
12. $\displaystyle\int \sin(ax)\cos(bx)dx = \frac{1}{b^2-a^2}\left[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)\right] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\displaystyle\int x^n \ln x\;dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, \quad x > 0$

14. $$\begin{aligned}\int p(x)\,e^{ax}dx &= \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}dx \\ &= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots \\ &\quad (+-+-\cdots) \qquad \text{(signs alternate)}\end{aligned}$$

15. $$\begin{aligned}\int p(x)\sin ax\;dx &= -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\;dx \\ &= -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots \\ &\quad (-++--++\cdots) \qquad \text{(signs alternate in pairs after first term)}\end{aligned}$$

16. $$\begin{aligned}\int p(x)\cos ax\;dx &= \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\;dx \\ &= \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots \\ &\quad (++--++--\cdots) \qquad \text{(signs alternate in pairs)}\end{aligned}$$

IV. Integer Powers of $\sin x$ and $\cos x$.

17. $\displaystyle\int \sin^n x\, dx = -\frac{1}{n}\left(\sin^{n-1} x\right)\cos x + \frac{n-1}{n}\int \sin^{n-2} x\, dx, \quad n \text{ positive}$

18. $\displaystyle\int \cos^n x\, dx = \frac{1}{n}\left(\cos^{n-1} x\right)\sin x + \frac{n-1}{n}\int \cos^{n-2} x\, dx, \quad n \text{ positive}$

19. $\displaystyle\int \frac{1}{\sin^m x}dx = \frac{-1}{(m-1)}\frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2} x}dx,\ m \neq 1,\ m \text{ positive}$

20. $\displaystyle\int \frac{1}{\sin x}dx = \int \csc x\, dx = \ln[\csc x - \cot x] + C$

21. $\displaystyle\int \frac{1}{\cos^m x}dx = \frac{1}{(m-1)}\frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2} x}dx. \quad m \neq 1, \quad m \text{ positive}$

22. $\displaystyle\int \frac{1}{\cos x}dx = \int \sec x\, dx = \ln\left[\sec x + \tan x\right] + C$

23. $\displaystyle\int \sin^m x \cos^n x\, dx:$ If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both m and n are even and negative is ommitted.

V. Quadratic in the Denominator

24. $\displaystyle\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$

25. $\displaystyle\int \frac{bx+c}{x^2+a^2}dx = \frac{b}{2}\ln\left|x^2+a^2\right| + \frac{c}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$

26. $\displaystyle\int \frac{1}{(x-a)(x-b)}dx = \frac{1}{a-b}\left(\ln|x-a| - \ln|x-b|\right) + C, \quad a \neq b$

27. $\displaystyle\int \frac{cx+d}{(x-a)(x-b)}dx = \frac{1}{a-b}\left[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|\right] + C, \quad a \neq b$

VI. Integrands involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

28. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C$

29. $\displaystyle\int \frac{1}{\sqrt{x^2 \pm a^2}}dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

30. $\displaystyle\int \sqrt{a^2 \pm x^2}dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}dx\right) + C$

31. $\displaystyle\int \sqrt{x^2 - a^2}dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}dx\right) + C$

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