

Math 249 L04, Fall 2003  
Work Sheet

1. Solve the following inequalities

(a)  $\frac{2}{5-x} \geq \frac{4}{x}$

(b)  $(x-4)(x+4) > 0$

2. Find the domain and the range of

(a)  $f(x) = \frac{x+2}{x-1}$

3. Find the domain of

$$f(x) = \sqrt{\frac{2}{x^2+3}}$$

4. Graph the function  $\sin(\frac{1}{4}x) + 1$

5. Find the following limits

(a)  $\lim_{x \rightarrow \infty} \sqrt[3]{\frac{5x+4}{2x-4}}$

(b)  $\lim_{x \rightarrow 0^+} \frac{\sin 3x}{1 - \cos 6x}$

(c)  $\lim_{h \rightarrow 0} \frac{\tan 4h}{\sin 8h}$

(d)  $\lim_{x \rightarrow 1^-} \frac{2x^2 + 3x + 1}{|2x^2 + 3x + 1|}$

6. Let  $f(x) = \begin{cases} -x^4 + 3 & \text{if } x \leq 2 \\ x^2 + 9 & \text{if } x > 2 \end{cases}$ . Is  $f$  continuous everywhere? Justify your conclusion.

7. Use intermediate value Theorem to show that the equation  $x^3 - 4x + 2 = 0$  has a solution on the interval  $[1, 2]$ .

8. Find the points of discontinuity if any of the following functions

(a)  $y = \frac{x-4}{x^2-16}$

(b)  $y = \frac{\sin \theta}{\theta}$

(c)  $y = \sqrt[3]{3x-1}$

9. Find  $\frac{dy}{dx}$  of the following

(a)  $\frac{1}{y} + \frac{1}{2x} = 1$

(b)  $y = (\sin(x^2))^{-\frac{1}{2}}$

(c)  $y = \tan(1 + \sin(x^3))$

(d)  $\sin(x^3y^3) = x^2 + x$

10. Two cyclists start moving from the same point. One travels south at 50km/h and the other travels west at 30km/h. At what rate is the distance between the two cyclists increasing two hours later?
- 11.
- (a) Use local linear approximation to confirm

$$\sqrt{1+x} \approx \frac{5}{4} + \frac{x}{4}$$

for points close to  $x_0 = 3$ .

- (b) Use a) to estimate  $\sqrt{4.2}$ .

## Solutions

1. **a.**  $(-\infty, 0) \cup \left[\frac{10}{3}, 5\right)$       **b.**  $(-\infty, -4) \cup (4, \infty)$
2. Domain  $\{x \mid x \neq 1\}$ ,      Range  $\{y \mid y \neq 1\}$
3.  $(-\infty, \infty)$
- 4.
5. **a.**  $\sqrt[3]{\frac{5}{2}}$       **b.**  $\infty$       **c.**  $\frac{1}{2}$       **d.** 1
6. No continuous
- 7.
8. **a.**  $x = 4$ ,      **b.**  $\theta = 0$ ,      **c.** No points of discontinuity.
9. **a.**  $\frac{dy}{dx} = -\frac{y}{2x^2}$       **b.**  $\frac{dy}{dx} = -x (\sin x^2)^{-\frac{3}{2}} \cos(x^2)$ .
- c.**  $\frac{dy}{dx} = \sec^2(1 + \sin(x^3)) (\cos(x^3)) (3x^2)$
- d.**  $\frac{dy}{dx} = \frac{2x + 1 - 3 \cos(x^3 y^3) x^2 y^3}{3x^3 y^2 \cos(x^3 y^3)}$
10.  $\frac{2300}{\sqrt{13600}} km/h$ .