Math 249, Fall 2003
PRACTICE PROBLEMS

1. Express the function $f(x)$ in piecewise form with out using absolute values $f(x) = 3|x-2| - |x+1|$

2. Find or simplify $\dfrac{f(x+h)-f(x)}{h}$ where $h \neq 0$ and $f(x) = 4x^2 - 5x + 7$.

3. Find the domain and the range of the function $g(x)$.

   (a) $g(x) = \sqrt{x^2 - 3x - 4}$

   (b) $g(x) = \dfrac{x^3 - 2x^2}{x - 2}$

4. Find the value of the constant k, if possible, that will make the function continuous every where.

$$f(x) = \begin{cases} kx^2 & x \leq 2 \\ 2x + k & x > 2 \end{cases}$$

5. Show that $f(x) = 5 - x - x^2$ has at least one solution on the interval $[-4, 0]$.

6. Find the following limits

   (a) $\displaystyle\lim_{x\to\infty} \frac{3x+4}{\sqrt{2x^2-5}}$

   (b) $\displaystyle\lim_{x\to 0} \frac{3x^2}{1-\cos^2(\frac{1}{2}x)}$

   (c) $\displaystyle\lim_{h\to\frac{\pi}{2}} \frac{1-\sin h}{\frac{1}{2}\pi - x}$ (Hint use $t = \frac{\pi}{2} - x$)

7. Find $\dfrac{dy}{dx}$ by implicit differentiation.

$$x \sin y + y \cos x = 1$$

8. Find $\dfrac{dy}{dx}$ for the following:

   (a) $y = \sqrt{3}^{x \sin x}$

   (b) $y = (\tan x)^{\ln x}$

9. At what point of the curve $xy = (1 - x - y)^2$ is the tangent line parallel to the $x-$axis?

10. Use local linear approximation to estimate $\cos(46°)$.

11. Solve for x the following equation $\ln\left(\dfrac{1}{x^2}\right) + \ln\left(4x^6\right) = \ln 4$

12. (a) Find the critical points of the function $f(x) = x^3 - 3x^2 + 3$ and determine if they are maximum, minimum or none.

    (b) Find the intervals where the function is increasing and decreasing.

    (c) Find the intervals where the function is concave up and concave down and the inflection points.

13. A rancher has 200feet of fencing with which he wants to enclose two adjacent regular corrals. What dimensions should be used so that the enclose area will be a maximum?

14. Solve the following integrals:

    (a) $\int \sin(2x)\cos(2x)dx$

    (b) $\int \frac{\csc^2 x}{\cot^3 x}dx$

    (c) $\int x^2\sqrt{1-x}dx$

    (d) $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right) dx$

    (e) $\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}dx$