MATH 249
Worksheet #1-Solutions

1. (a) Solve for x: $|2x+1| \le |x-2|$

   (b) Solve for x: $\dfrac{3}{x+1} > \dfrac{1}{3}$.

   For 1 a)

   Since $|...|$ is always positive or zero we can square both sides and the sign of the inequality

   stays the same: $(2x+1)^2 \le (x-2)^2$ since $|...|^2 = (...)^2$Now

   $4x^2 + 4x + 1 \le x^2 - 4x + 4$,everything on one side: $3x^2 + 8x - 3 \le 0$

   $(3x-1)(x+3) \le 0$,thus split points are: $x = -3, \frac{1}{3}$,

   testing: $- -^{pos} - - -_{3} - -^{neg} - -_{\frac{1}{3}} - -^{pos} - --$

   so the solutions set :$\left[-3, \frac{1}{3}\right]$

   b)

   $x \neq -1$

   everything on one side and common denominator: $\dfrac{3 \cdot 3 - (x+1)}{(x+1)3} > 0$,simplify:

   $\dfrac{9-x-1}{3(x+1)} > 0$ then $\dfrac{8-x}{(x+1)(3)} > 0$.So split points are : $x = 8, -1$

   testing: $- -^{neg} - - -_{-1} - -^{pos} - -_{8} - -^{neg} - --$

   solution set: $(-1, 8)$.

2. Find the radius and centre of the circle $x^2 + 4x + y^2 - 2y = 11$.

   For 2)

   Complete the squares: $(x+2)^2 - 4 + (y-1)^2 - 1 = 11$ so $(x+2)^2 + (y-1)^2 = 16$

   thus $r = 4$ and the point $(-2, 1)$ is the centre.

3. Solve for x:

   (a) $|x+1| + 2 > 0$

   (b) $\dfrac{3}{x+1} \ge \dfrac{2}{x+3}$.

   For 3a)

   Since $|...|$ is always positive or zero $|x+1| + 2$ is always positive ,so solution set $(-\infty, +\infty)$

   b)

   $x \neq -1, -3$

everything on one side and common denominator: $\dfrac{3(x+3)-2\,(x+1)}{(x+1)(x+3)} \geq 0$,simplify:

$\dfrac{3x+9-2x-2}{(x+1)\,(x+3)} \geq 0$ then $\dfrac{(x+7)}{(x+1)\,(x+3)} \geq 0$.So split points are : $x = -7, -3, -1$

testing: $\quad -^{neg} -_{-7} - - -^{pos} - - -_{-3} - -^{neg} - -_{-1} - -^{pos} - - -$

solution set: $[-7, -3) \cup (-1, +\infty)$

4. Given four lines $l_1 : 3x + 2y = 1$ $l_2 : 2y - 3x = 0$ $l_3 : 3x - 2y = 0$ and $l_4 : 2x - 3y = 2$ choose all which are

   (a) parallel
   (b) perpendicular.

   For 4)

   Find slopes: $m_1 = -\frac{3}{2}$,$m_2 = \frac{3}{2}, m_3 = \frac{3}{2}, m_4 = \frac{2}{3}$ so $l_2 \|\ l_3$ since they have the same slope

   and $l_1 \perp l_4$ since $m_1 \cdot m_4 = -1$.

5. Solve for x:

   (a) $\dfrac{1}{x+1} \leq 1 + x$
   (b) $|3x - 2| > 0$.

   For 5 a)

   $x \neq -1$

   everything on one side and common denominator: $\dfrac{1-(x+1)^2}{(x+1)} \leq 0$,simplify:

   $\dfrac{1-x^2-2x-1}{(x+1)} \leq 0$ then $\dfrac{-x\,(x+2)}{(x+1)} \leq 0$.So split points are : $x = 0, -2, -1$

   testing $\quad -^{pos} - -_{-2} - -^{neg} -_{-1} - -^{pos} - - -_{0} - -^{neg} - -$

   solution set: $[-2, -1) \cup [0, +\infty)$

   b)

   Since $|...|$ is always positive or zero we have to elliminate zero :$3x - 2 = 0$ for $x = \frac{2}{3}$

   The solutions : $x \neq \frac{2}{3}$ or $\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, +\infty\right)$

6. Find an equation of the line perpendicular to the x-axis passing through the point $(-1, 3)$.

   For 6)

   $\perp$ to x-axis means a vertical line so $x = -1$ (y is any).

7. Solve for x:

   (a) $3x + 7 > x^2$

   (b) $\frac{x}{2} < \frac{2}{x+3}$.

For 7 a)

Everything on one side: $0 > x^2 - 3x - 7$ now find the roots ,first

discriminant $D = (-3)^2 - 4 \cdot 1 \cdot (-7) = 9 + 28 = 37$, so using the formula roots are

$x_1 = \frac{3-\sqrt{37}}{2} = -1.54$ and $x_2 = \frac{3+\sqrt{37}}{2} = 4.54$

Now testing : $- -^{pos} - -_{x_1} - -^{neg} - - -_{x_2} - -^{pos}-$

OR parabola open up is below the x-axis if $x \in (x_1, x_2) = (-1.54, 4.54)$.

b)

$x \neq -3$

everything on one side and common denominator: $\dfrac{x(x+3) - 2 \cdot 2}{2(x+3)} < 0$,simplify:

$\dfrac{x^2 + 3x - 4}{2(x+3)} < 0$ then $\dfrac{(x+4)(x-1)}{2(x+3)} < 0$.So split points are : $x = -4, -3, 1$

(a)

testing:$-^{neg} - -_{-4} - -^{pos} - -_{-3} -^{neg} - -_{1} -^{pos} - -$

solution set: $(-\infty, -4) \cup (-3, 1)$.

8. Which of the given circles has bigger radius

$$x^2 - 6x + y^2 = 7 \text{ or } x^2 + y^2 + 2y = 15 \text{ ?}$$

For 8)

Complete squares : $(x-3)^2 - 9 + y^2 = 7, x^2 + (y+1)^2 - 1 = 15$ SO the equations are: $(x-3)^2 + y^2 = 16, x^2 + (y+1)^2 = 16$ thus radii are the same $r = 4$,
the centres are points $(3, 0)$ and $(0, -1)$.