FINAL Handout
MATH 249

1. Evaluate the limits:

   (a) $\displaystyle\lim_{x\to\pi} \frac{\cos\left(\frac{x}{2}\right)}{\pi - x}$ (b) $\displaystyle\lim_{x\to-\infty} \frac{\cos\left(\frac{x}{2}\right)}{\pi - x}$

   (c) $\displaystyle\lim_{x\to+\infty} \left(x2^{-x^2}\right)$ (d) $\displaystyle\lim_{x\to-\infty} \frac{x}{\sqrt{4x^2+3x+7}}$

2. Find the domain and the derivative of f of

   (a) (a) $f(x) = \frac{x}{3} e^{-\sin\left(\frac{3}{x}\right)}$ (b) $f(x) = \frac{\ln(2x-3)}{e^{-x^2}}$

3. **A** Sketch the graph of $y = e^{2x}(6x^2 - 2x - 1)$ i.e.

   (a) find the domain, range, vertical and horizontal asymptotes, x and y intercepts;

   (b) find the intervals where f is increasing or decreasing;local extrema;

   (c) find the intervals where f is concave down or up

   B

   Sketch the graph of $y = x(4-x)^3$.Indicate where the function is increasing,decreasing, concave up,concave down;find the domain and range.

4. (a) Find the tangent approximation (linearization) of $f(x) = \frac{1}{\sqrt{2x^2+1}}$ around $x_0 = 2$.

   (b) Use it to estimate $\frac{1}{\sqrt{3}}$.

5. **A** Sketch a graph of <u>one</u> function f satisfying all the following conditions:

   (a) f is defined on $]-\infty, +\infty[$, continuous there except

   (b) f is discontinuous at $x = 2, 4$ where $\lim_{x\to 4^-} f(x) = f(4) = 0$, $x = 2$ is a vertical asymptote.

   (c) $y = 3$ is a horizontal asymptote and $\lim_{x\to-\infty} f(x)$ <u>does not exist</u>,

   (d) f is increasing on $]3,4[$ and on $]4,+\infty[$, f is decreasing on $]0,2[$ and on $]2,3[$, and $f'(x) = 0$ for all $x \in]-2,0[$;

   (e) f is concave up on $]0,1[$ and on $]3,4[$; f is concave down on $]1,2[$, on $]2,3[$ and on $]4,+\infty[$;

   (f) absolute maximum value is 6, and local minimum value is -2.

   B Sketch a graph of <u>one</u> function f satisfying all the following conditions:

   (a) f is defined on $]0,\infty[$

(b) f is discontinuous at $x = 1, 2, 3$ where $\lim_{x\to 2} f(x) = 3$, $\lim_{x\to 3} f(x)$, DNE (does not exist).

(c) $x = 1$ is V.A., $y = 2$ is H.A.

(d) f is increasing on the intervals $]2, 3[$ $]3, 4[$ $]5, \infty[$
f is decreasing on $]0, 1[$ and on $]4, 5[$
$f'(x) = 0$ for all $x \in (]1, 2[$

(e) f is concave up on the intervals $]2, 3[$ and $]4, 6[$, concave down on $]3, 4[$ and on $]6, \infty[$

(f) absolute maximum value is 5, local minimum value is -1.

C Sketch a graph of <u>one</u> function f satisfying all the following conditions:

(a) f is defined on $[-1, +\infty[$ continuous there except

(b) f is discontinuous at $x = 1, 3$ where $\lim_{x\to 1} f(x)$ <u>does not exist</u>,.

(c) $x = 3$ is a vertical asymptote,and $y = 2$ is a horizontal asymptote,

(d) f is increasing on $]-1, 0[$ and on $]3, +\infty[$, f is decreasing on $]0, 1[$ and on $]2, 3[$, and $f'(x) = 0$ for all $x \in]1, 2[$;

(e) f is concave up on $]-1, 0[$,on $]0, 1[$and on $]3, 4[$, f is concave down on $]2, 3[$ and on $]4, +\infty[$;

(f) absolute maximum value is 7, and local minimum value is 0.

6. **A**

A box with a square base(bottom) and NO top(lid) has a volume of 9 m^3.Find the dimensions of the most economical box

if the material for the base costs \$2 per m^2 and the material for the sides \$3 per m^2.

B

A landscape architect plans to enclose a 280 m^2 rectangular region in a botanical garden.

She will use shrubs costing \$25.00 per meter along three sides and fencing

costing \$10.00 per meter along the fourth side.

Find the dimensions of the region to minimize the total cost.

7. Find

(a) $\displaystyle\int \frac{3\sqrt{x}-5}{x\sqrt{x}}dx$ (b) $\displaystyle\int 2x^3\sqrt{2x^2+3}dx$ (c) $\displaystyle\int \sin\frac{x}{3}dx$

in the domain of definition.

8. Evaluate

(a) $\displaystyle\int_2^3 x2^{x^2}dx$ (b) $\displaystyle\int_0^1 \frac{4x+3}{3-2x}dx$ (c) $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}dx$