

MATHEMATICS 221 L20 WINTER 2002
MIDTERM EXAMINATION

Friday, March 15, 2002

Duration: 50 minutes.

[8] 1. Solve the system:

$$\begin{array}{rcccccccl} x & - & 2y & + & z & & & = & 5 \\ 2x & - & 4y & + & z & + & 2w & = & 8 \\ -4x & + & 8y & - & z & - & 6w & = & -14 \end{array}$$

[5] 2. (a) Prove that if A is an $n \times n$ invertible matrix then $\det(\text{adj} A) = (\det A)^{n-1}$.

[5] (b) Let A be a 3×3 invertible matrix. Find $\det[-2A^2(\text{adj} A)^{-1}]$.

[8] 3. Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 4 & -3 \end{bmatrix}$.

(a) Find an invertible matrix U so that $UA = B$.

(b) Express U^{-1} as a product of elementary matrices.

[6] 4. Find all values of x so that the following matrix is **not** invertible:

$$\begin{array}{ccc} 1 & -x & -x \\ x & -2 & -x \\ x & x & -3 \end{array}$$

[6] 5. Express $(\sqrt{3} + i)^{-4}$ in the form $a + bi$ where a and b are real numbers.

[8] 6. Let $A = \begin{bmatrix} 3 & 6 \\ -2 & -4 \end{bmatrix}$.

(a) Is A diagonalizable? If A is diagonalizable, find an invertible matrix P and a diagonal matrix D so that $A = PDP^{-1}$.

(b) Compute A^{10} .