



Applied Mathematics 503

The Mathematics of Wavelets, Signal and Image
Processing

(see Course Descriptions for the applicable academic year: <http://www.ucalgary.ca/pubs/calendar/>)

Syllabus

<u>Topics</u>	<u>Number of Hours</u>
Review of linear algebra for inner product spaces: inner product, norms, linear subspaces, linear independence, orthogonal and orthonormal bases, Gram-Schmidt, convergence in norm, infinite orthogonal bases	3
Fourier series for real-valued functions on an interval: definition and properties, uniform and pointwise convergence, Parseval's theorem, L2 convergence	6
Discrete and Fast Fourier Transforms (transforms on a finite cyclic group): forward and inverse transform, cyclic convolution, convolution theorem, 2D transforms	6
Fourier transform on the real line: definition and properties, Riemann-Lebesgue lemma, inverse transform, convolutions and time-invariant systems, Plancherel's theorem	6
Representing signals: on the real line, discretely sampled, A/D and D/A conversion, the Shannon sampling theorem, aliasing, time-invariant filtering, convolution and the Fourier transform	6
Introduction to wavelets: the Haar wavelet, scaling function and wavelet function. Haar decomposition and reconstruction, multiresolution analysis. Compactly supported wavelets	9
TOTAL HOURS	36

AMAT 503 Course Outcomes

AMAT 503- The mathematics of wavelets, signal and image processing.

The overall goal is to develop a basic understanding and expertise in applying the modern mathematical ideas of wavelets to real applications of sound and image processing.

1. The student will understand how a physical sound, visual image, or movie is represented in the computer as an ordered vector or array of numbers. The student will develop both an intuitive and quantitative understanding of the physical and numerical limitations of such a representation.
2. The student will be familiar with representing these sounds and images in various encodings used on real systems, included linear and compressed amplitude encoding for sound, grey scale and colour maps for images.
3. The student will be able to apply mathematical ideas from linear algebra (matrix transformations, convolution, inner product and norm calculations, orthogonal basis, infinite dimensions and convergence) to real applications in sound and image processing.
4. The student will review the properties of the Fourier transform and become familiar with its application for fast and accurate algorithms applied to sounds and images.
5. The student will be able to represent physical signals using a variety of wavelet transforms, from the basic Haar wavelet, to Daubechies breakthrough approach with smooth, compactly supported orthogonal wavelets. The student will understand the utility of this novel representation in real communications, as well as in sound and image processing systems. The student will gain an appreciation for the deep mathematical theory behind these wavelet transforms.
6. Through specific projects, the student will gain the ability to code these various mathematical methods on a digital computer, gaining a hands-on experience in working with real sounds and images on the computer. They will become familiar with the advanced algorithms that form the basis for popular tools such as PhotoShop, medical CAT scans and seismic imaging processing software.
