University of Calgary Department of Mathematics and Statistics
Amat 433 L01 Nov. 29, 2002 Midterm Test II. Duration: 50 minutes
No Calculators Allowed

The marks for each problem are shown in brackets. Total marks = 80.

1. Let $f(z) = \dfrac{z}{(z-2)(z-3)} = \dfrac{3}{z-3} - \dfrac{2}{z-2}$.

Find the value of $\int_C f(z)\,dz$ when:

[5] (a) C = circle, $|z| = 4$.

[10] (b) C = half circle, $|z| = 1$, $z = e^{i\theta}$, $0 \le \theta \le \pi$.

[15] 2. Use complex analysis to find the (principal) value of the real integral $\int_{-\infty}^{\infty} \dfrac{\cos x}{x^4 - 1}\,dx$.

3. Let $f(z) = \dfrac{z}{z-3}$. Find the Laurent series of $f(z)$:

[5] (a) About $z_0 = 3$, convergent for $|z-3| > 0$

[5] (b) About $z_0 = 0$, convergent for $|z| > 3$.

[10] 4. Use complex analysis to find the value of the real integral $\int_0^{2\pi} \dfrac{d\theta}{\sqrt{5} + \sin\theta}$

[10] 5. Consider linear operators from the real vector space R^2 to the real vector space R^3. Let the linear operator T be defined by

$$T\vec{v} = (x, x+y, x-y)$$

where $\vec{v} \in R^2$, $\vec{v} = (x, y)$, x and y are any real numbers. Find the matrix representation of T in the standard bases of R^2 and R^3.

Amat 433 Fall 2002 Midterm II. Answers

1(a) $2\pi i$

1(b) $\ln(8/9)$

2. $\frac{-\pi}{2e} - \frac{\pi}{2}\sin 1.$

3(a) $1 + \frac{3}{z-3}$

3(b) $1 + \frac{3}{z} + \frac{9}{z^2} + \frac{27}{z^3} + \ldots$

4. π.

5. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}$