GRADUATE COURSES Note: In addition to the prerequisites listed below, consent of the Applied Mathematics Department or the Pure Mathematics Department is a prerequisite for these graduate courses.

Mathematics Research Seminar 600
A professional skills course, focusing on the development of technical proficiencies that are essential for students to succeed in their future careers as practicing mathematicians in academia, government, or industry. The emphasis is on delivering professional presentations and using modern mathematical research tools. A high level of active student participation is required.
Course Hours: Q(3S-0)
MAY BE REPEATED FOR CREDIT NOT INCLUDED IN GPA

Mathematics Measure and Integration 601
Abstract measure theory, basic integration theorems, Fubini's theorem, Radon-Nikodym theorem, Lp spaces, Riesz representation theorem.
Course Hours: H(3-0)
Prerequisite(s): Mathematics 545 or Pure Mathematics 545 or consent of the Department.
Antirequisite(s): Credit for more than one of Mathematics 501, 601, Pure Mathematics 501 and 601 will not be allowed.

Mathematics Analysis III 603
Sequences and series of functions; Lebesgue integration on the line, Fourier series and the Fourier transform, pointwise convergence theorems, distributions and generalized functions.
Course Hours: H(3-0)
Prerequisite(s): Mathematics 447 or a grade of "B+" or better in Pure Mathematics 445 or Mathematics 445.
Antirequisite(s): Not open to students with credit in Mathematics 545 or Pure Mathematics 545.

Mathematics Differential Equations III
605
Course Hours: H(3-0)
Prerequisite(s): *Applied Mathematics 411 and Pure Mathematics 445 or 545 or equivalents.*

Mathematics Algebra III
607
(formerly Pure Mathematics 611)
This course provides a sophisticated introduction to modules over rings, especially commutative rings with identity. Major topics include: snake lemma; free modules; tensor product; hom-tensor duality; finitely presented modules; invariant factors; free resolutions; and the classification of finitely generated modules over principal ideal domains. Adjoint functors play a large role. The course includes applications to linear algebra, including rational canonical form and Jordan canonical form. Students who successfully complete this course will be prepared to take advanced graduate courses in mathematics that rely on algebraic structures arising from rings and modules.
Course Hours: H(3-0)
Prerequisite(s): *Pure Mathematics 431 or Mathematics 411; or consent of the Department. Pure Mathematics 431 is recommended.*
Antirequisite(s): Credit for more than one of Pure Mathematics 511, 611 and Mathematics 607 will not be allowed.

Mathematics Functional Analysis
617
(formerly Applied Mathematics 617)
Introduction to some basic aspects of Functional Analysis, Hilbert and Banach spaces, linear operators, weak topologies, and the operator spectrum.
Course Hours: H(3-0)
Prerequisite(s): *Mathematics 545 or Mathematics 603.*
Antirequisite(s): Credit for more than one of Applied Mathematics 617 and Pure Mathematics 617 and Mathematics 617 will not be allowed.

Mathematics Complex Analysis
621
Course Hours: H(3-0)
Prerequisite(s): *Mathematics 335 or 355 or Pure Mathematics 435 or 455; or consent of the
**Mathematics Introduction to Algebraic Topology 625**

This is an introduction to the algebraic invariants that distinguish topological spaces. Specifically, the course focuses on the fundamental group and its applications, and homology. Students will also be introduced to the basics of homological algebra.

Course Hours: H(3-0)

Prerequisite(s): *Pure Mathematics 505 and Pure Mathematics 431, or consent of the Department.*

**Mathematics Algebraic Geometry 627**

The objective of this course is to provide an introduction to modern algebraic geometry sufficient to allow students to read research papers in their fields which use the language of schemes. Topics will include Spectra of rings; the Zariski topology; affine schemes; sheaves; ringed spaces; schemes; morphisms of finite type; arithmetic schemes; varieties; projective varieties; finite morphisms, unramified morphisms; etale morphisms.

Course Hours: H(3-0)

Prerequisite(s): *Mathematics 607 or consent of the Department.*

**Mathematics Discrete Mathematics 631**

This pair of courses will cover two major fields of Discrete Mathematics. Topics covered in Discrete Geometry will include Euclidean, spherical and hyperbolic n-spaces, trigonometry, isometries, convex sets, convex polytopes, (mixed) volume(s), classical discrete groups, tilings, isoperimetric inequalities, packings, coverings.

The second course will present an introduction to graph theory at an advanced level: connectivity; trees; Euler trails and tours; Hamilton cycles and paths; matchings; edge colourings; vertex colourings; homomorphisms; plane and planar graphs; extremal graph theory and Ramsey theory. Additional topics
if time permits.

631.01 Discrete Geometry
631.03 Graph Theory

Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.

Mathematics Geometry of Numbers
635

At the heart of the geometry of numbers is the interplay of the group-theoretic notion of lattice and
the geometric concept of convex set, the lattices representing periodicity, the convex sets geometry.

Proposed topics to be covered include convex bodies and lattice points, the critical determinant, the
covering constant and the inhomogeneous determinant of a set, Star bodies, methods related to
the above, and homogeneous and inhomogeneous forms.
Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.

Mathematics Infinite Combinatorics
637

The course is an excursion into the infinite world, from Ramsey Theory on the natural
numbers, to applications in Number Theory and Banach Spaces, introduction to tools in Model
Theory and Logic, fascinating homogeneous structures such as the rationals and the Rado
graph, and possibly further explorations into the larger infinite world.
Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.

Mathematics Number Theory
641

Number Theory is a diverse and important topic in modern mathematics. It is distinguished by a
very distinct dichotomy in its main research focuses. Both of these two distinct perspective will
be examined in these courses.

Algebraic Number Theory will provide a standard introduction to the subject. This will include
an introduction to number fields, rings of integers, ideals, unique factorization, the different and
the discriminant. The main objective to the course will be to prove the finiteness of the class
number and Dirichlet's Unit Theorem.
Analytic Number Theory will focus on developing tools that aid in studying the average behavior of functions of interest from a number theoretic perspective. Topics covered will include arithmetic functions, Dirichlet series, Dirichlet convolution, Abel summation, Riemann-Stieltjes integration, the Riemann zeta function, and L-functions. The ultimate goal of the course is a complete proof of the Prime Number Theorem.

651.01 Algebraic Number Theory
651.03 Analytic Number Theory
Course Hours: H(3-0)
Prerequisite(s): consent of the Department.
Note: Mathematics 607 is recommended for Mathematics 651.01, but not required. Mathematics 421 or equivalent is recommended for 651.03.

Mathematics Computational Number Theory
643
(formerly Pure Mathematics 627)
An investigation of major problems in computational number theory, with emphasis on practical techniques and their computational complexity. Topics include basic integer arithmetic algorithms, finite fields, primality proving, factoring methods, algorithms in algebraic number fields.
Course Hours: H(3-0)
Prerequisite(s): Pure Mathematics 427 or 429; or consent of the Department.
Antirequisite(s): Credit for more than one of Pure Mathematics 527, 627 and Mathematics 643 will not be allowed.

Mathematics Modular Forms
647
Modular forms and automorphic representations and their L-functions occupy a central role in number theory. The overarching goal of both courses is to explain the Modularity Theorem, from two perspectives.

Classical Perspective on Modular Forms will introduce the student to modular curves as moduli spaces for elliptic curves and, after introducing modular forms as differential forms on modular curves and studying their Fourier expansion, will construct Eisenstein series and the algebra of Hecke operators. The course culminates in a study of L-functions attached to modular forms and the modularity theorem.

An Introduction to Automorphic Representations is an introduction to the Langlands Programme. After explaining the adelic perspective and reviewing some elementary class field theory, the course
introduces the adelic integral forms of Hecke operators on a Hilbert space, leading almost directly to the definition of automorphic representations and their Satake parameters. The course culminates in a study of partial L-functions attached to automorphic representations and known instances of the Langlands Correspondence.

647.01 Classical Perspective on Modular Forms
647.03 An introduction to Automorphic Representations

Course Hours: H(3-0)
Prerequisite(s): Mathematics 607; or consent of the Department.

Mathematics Topics in Applied Mathematics
651
(formerly Applied Mathematics 601)
Topics will be chosen according to the interest of the instructors and students.
Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.

MAY BE REPEATED FOR CREDIT

Mathematics Topics in Pure Mathematics
653
(formerly Pure Mathematics 603)
Topics will be chosen according to the interest of the instructors and students.
Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.

MAY BE REPEATED FOR CREDIT

Mathematics Scientific Modeling & Computation 1
661
This group of courses will address some core topics related to scientific modeling and computation.

Optimization is ubiquitous in modern science and engineering fields. The Convex Optimization course will provide an introduction to modern convex optimization, including basics of convex analysis and duality, linear conic programming, robust optimization, and applications.

Modern scientific computing is at the forefront of the computationally assisted discovery that propels
many advanced science and engineering disciplines. Scientific Computation course will focus on both the methodological and the implementation components underlying the modern scientific computations with the natural emphasis on linear algebra. Particularly, this will include such fundamental topic as iterative and elimination methods for linear systems, the numerical eigenvalue problem, condition number and the general notion of conditioning, parallel computation, the fast Fourier transform, as well as some components of the modern computing architecture and its implications for the numerical algorithms.

Differential Equations (DE) is a long established domain of applied mathematics with numerous applications in physics, biology, engineering, etc. The Numerical Differential Equations course primarily addresses fundamentals of solving DEs numerically addressing the existence, stability and efficiency of such methods. Topics to be covered include Euler’s method, multistep methods, Runge-Kutta methods and the shooting method for ordinary DEs. finite difference methods, higher-order finite difference methods, the finite element method, finite volume methods, spectral methods, boundary element methods and multigrid methods for linear and nonlinear elliptic, parabolic and hyperbolic equations and the characteristic method for first order partial differential equations.

661.01 Convex Optimization
661.03 Scientific Computation
661.05 Numerical Differential Equations

Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.
Note: Mathematics 603 is recommended for Mathematics 661.01
663.05 Differential Equations

Course Hours: H(3-0)
Prerequisite(s): 2 of Mathematics 601, 603 and 605; or consent of the Department.
Note: Mathematics 601, 603 and 605 is recommended. Additionally, Mathematics 661.01 and Mathematics 617 are recommended for Mathematics 663.01

Mathematics Introduction to Quantum Information 667

An introductory course to quantum information science focusing on the mathematical treatment of a broad range of topics in quantum Shannon theory that are applicable also to quantum cryptography, quantum computing, and quantum resource theories.

Topics include quantum states, quantum channels, quantum measurements, completely positive maps, Neumarkís theorem, Stinespring dilation theorem, Choi-Jamiolkowski isomorphism, the theory of majorization and entanglement, the Peres-Horodecki criterion for separability, Shannon’s noiseless and noisy channel coding theorems, Lieb’s theorem and the strong subadditivity of the von Neumann entropy, Schumacher’s quantum noiseless channel coding theorem, and the Holevo-Schumacher-Westmoreland theorem.

Course Hours: H(3-0)
Prerequisite(s): Applied Mathematics 411 or Physics 443; or consent of the Department.

Mathematics Scientific Modeling & Computation 2 669

Further topics related to scientific modeling and computation will be addressed in these courses.

Wavelet transforms may be viewed as a generalization of Fourier transforms, and have wide range of applications in imaging, signal processing, etc. The Wavelet Analysis course covers the design and implementation of wavelet methods for modern signal processing, particularly for one- and two-dimensional signals (audio and images). Topics include discrete Fourier transforms, filter banks, orthogonal and biorthogonal wavelets, and second generation wavelets.

The Mathematical Biology course will introduce student to discrete models of mathematical biology, including difference equations, models of population dynamics and the like. Additional topics will include stability of models describe by difference equations, continuous spatially homogeneous processes and spatially distributed models. Additional topics will be covered as time permits.

667.01 Wavelet Analysis
667.03 Mathematical Biology

Course Hours: H(3-0)
Prerequisite(s): *Mathematics 617 is a required for Mathematics 667.01; or consent of the Department. Consent of the Department for Mathematics 667.03.*
Diffusion processes, The Feynman-Kac formula, Kolmogorov backward/forward equations, Dynkin’s formula

Course Hours: H(3-0)
Prerequisite(s): Consent of the Department.

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Mathematics Advanced Mathematical Finance 1
691

These courses cover more specific areas of mathematical finance, and are intended to follow on from Mathematics 681.

Lévy processes generalize the diffusion processes used in Mathematics 681 and enable the development of more powerful models and derivative valuation techniques. 691.01 Lévy Processes (LP) will familiarize the student with Infinite divisibility, the Lévy-Khintchine formula; examples of LP; Poisson integration, the Lévy-Itô decomposition, subordinators; Markov processes, semi-groups and generators of LP; the Itô formula for LP, quadratic variation; LP as time-changed Brownian motion, change of measure (Girsanov theorem); stochastic differential equations driven by LP; the Feynman-Kac formula and martingale problem for LP; applications of LP; simulation of LP.

691.03 Credit Risk is intended for students who would like to learn about corporate bond markets. A corporate bond is a type of a fixed income security similar to a treasury bond, that is, it promises a fixed amount of cash flow or lump payment at maturity. However unlike in a treasury bond, the promised payments are subject to bankruptcy risk of the underlying firm. Hence modeling the bankruptcy risk of a firm is essential for understanding how corporate bonds are priced.

691.01 Lévy Processes
691.03 Credit Risk

Course Hours: H(3-0)
Prerequisite(s): Mathematics 681; or consent of the Department.

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Mathematics Advanced Mathematical Finance 2
693

These courses cover more specific areas of mathematical finance, and are intended to follow on from both Mathematics 681 and Mathematics 683.

Monte Carlo Methods for Quantitative Finance will consider computing derivative values and portfolio risk measures as expectations; random number generation, simulation of stochastic differential equations; variance reduction techniques; quasi-Monte Carlo methods; computing sensitivities; other
price processes; early exercise options; applications to risk management; Markov Chain Monte Carlo.

Energy, Commodity and Environmental Finance will introduce the student to energy and commodity markets; spot, futures, forwards and swap contracts; the theory of storage; models for spot prices, forward prices, electricity prices; model calibration; emissions market modelling; weather derivatives; energy risk management; valuation of spread options and real options.

693.01 Monte Carlo Methods for Quantitative Finance  
693.03 Energy, Commodity and Environmental Finance

Course Hours: H(3-0)  
Prerequisite(s): Mathematics 681 and 683 or consent of the Department.

In addition to the numbered and titled courses shown above, the department offers a selection of advanced level graduate courses specifically designed to meet the needs of individuals or small groups of students at the advanced doctoral level. These courses are numbered in the series 800.01 to 899.99. Such offerings are, of course, conditional upon the availability of staff resources.